REMARKS

ON

MATHEMATICAL OR DEMONSTRATIVE REASONING:

ITS CONNEXION WITH LOGIC;

AND ITS APPLICATION TO SCIENCE,

PHYSICAL AND METAPHYSICAL,

WITH REFERENCE TO SOME RECENT PUBLICATIONS.

BY

EDWARD TAGART, F.G.S.

MINISTER OF THE CHAPEL IN LITTLE PORTLAND-STREET, REGENT-STREET.

"The light of human minds is perspicuous words; but by definition first snuffed, and purged from ambiguity."—*Hobbes.*

"It is an old remark that geometry is the best logic."—*Berkeley's Analyst.*

LONDON:

JOHN GREEN, 121, NEWGATE STREET.

1837.

PRINTED BY RICHARD KINDER,
GREAT NEW STREET, FETTER LANE.

TO

THE REV. W. TURNER, JUN.,

OF HALIFAX.

My Dear Sir,

To your valued instructions in Manchester College, York, I am indebted for much of the pleasure which I have occasionally derived from metaphysical inquiries and discussions. In the metaphysical as well as theological department of that Institution, we were taught to study with care and candour the best works, not to cavil and confute, "nor to believe and take for granted, but to weigh and consider." To you, therefore, as one well skilled in weighing arguments and detecting fallacies, I beg leave to inscribe this little volume, and remain

Your grateful pupil,

EDWARD TAGART.

Bayswater,
September 1837.

PREFACE.

The remarks here offered to the reader were commenced some time ago, with the design of adapting them to the pages of a Review or Magazine. Hence they have perhaps too much of a critical and controversial air for a distinct Essay on an abstract subject. But the remarks grew under my hands; and becoming, as they grew, less and less fit for any existing periodical, they are now presented, but with great diffidence, in

a separate form—with what advantage, the reader of course will judge.

The well-instructed student of mathematics, of logic, of the nature and theory of language, and of what is called moral evidence, will be apt to remark at the close of the work, that it makes no specific addition to the amount of his knowledge. But if the views presented even to him be admitted to be correct as far as they go; if some thoughts are here conveniently brought together which must at least be sought for in widely-scattered sources; if they have the good effect of awakening the attention of the less profound to important points connected with the subject before him, hitherto overlooked; if they suggest or stimulate inquiries worthy of continuance,—the publication will not be in vain. Much use has been made of the sentiments

of others, so as to form a sort of philosophical discussion in which many authors are made to speak for themselves. But an ample apology for this, if one be necessary, will be found in the words of Dr. Law in his preface to the translation of Dr. King's Essay on the Origin of Evil :—

"A writer often does more good by showing the use of some of those many volumes which we have already, than by offering new ones, though this be of much less advantage to his own character. I determined therefore not to say anything myself where I could bring another conveniently to say it for me; and transcribed only so much from others as was judged absolutely necessary to give the reader a short view of the subject, and by that sketch to induce those who have leisure, opportunity, and inclination to go

further and consult the originals, and to afford some present satisfaction to those who have not.

"But how judiciously this is performed, the notes themselves must testify."

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REMARKS.

It is more than a century since Locke conceived and maintained, after Hobbes and perhaps others, that demonstrative reasoning was applicable to other subjects besides the mathematics, and particularly to morality. Doctors Law and Hartley, the disciples and successors of Locke, entered fully into his views; and Dr. Hartley especially was fond of exhibiting his reasoning in a mathematical form, and in some instances has very happily applied algebraic formulæ to illustrate, I do not say to confirm, his trains of moral speculation.

Mr. Whewell's Thoughts on Mathematics, in which he affirms that mathematics afford the best example of practical logic, and the elaborate article in the Edinburgh Review,

No. 126, on Mathematical Studies, which treats generally of the influence of mathematics upon the intellectual character and powers, have in some degree recalled attention to the subject, and induced me to offer some thoughts upon it, which I trust will not appear altogether unworthy of perusal.

I do this with earnestness, and even anxiety; not because I conceive that anything original or remarkable will be found in the following observations; for I bear in mind an aphorism of Dr. Johnson, "He who tries to say that which has never been said before him, will probably say that which will never be repeated after him;" but because clear and just views on this subject have a close and important bearing upon the pursuit of science of all kinds, whether physical or metaphysical; upon the attainment and diffusion of truth; upon the mental and moral improvement, and consequently the harmony and happiness, of man. These clear and just views appear to be absent from the minds and writings of many whose names are of no small account in the literary and scientific

world, although within easy reach of the inquiring, if they will use the glass supplied by the plain and manly writers of the true English school of philosophy.

I venture upon it further, because Mr. Whewell, in the second edition of his pamphlet, declines going more at length into the matters touched upon by the reviewer. He has therefore left the field open to any one who may dare to enter the lists against that formidable and heavily-armed knight.

The interest which may have been felt in the papers alluded to has perhaps already subsided, but the subject to which they relate is of permanent importance. Mr. Whewell discussed the relative value of different modes of pursuing mathematical studies, assuming their usefulness and importance. The reviewer, however, went into a much wider field, namely, the influence of mathematical studies upon the mental powers and character in general. And it was his strain of remark, so far as it was of a metaphysical character, his observations about "two logics," "dissimilar de-

velopments of thought," and "higher and lower faculties," which induced me to review the reviewer, and put together a few thoughts upon the nature of reasoning and evidence in general. With many of the positions of the reviewer and of those whom he quotes, about mathematical evidence and mathematical processes, the matter of this Essay will be found substantially to coincide; but if there be any truth in his view of the disqualification of mere mathematicians for inquiries into mental and moral philosophy, I have endeavoured to approach more closely to the sources of that disqualification, or rather so to point out the distinction between the nature of our thoughts and language on mathematical and other subjects, as to furnish some useful guidance to reasoners and inquirers in morals and religion.

Moreover, while it seemed to me that the reviewer had merely taken occasion from Mr. Whewell's pamphlet to propound certain semi-German notions, and heap together certain Kantian phrases, than which none are more utterly distasteful to a healthful

English palate, it is perceptible, if I mistake not, that Mr. Whewell himself is not altogether so clear and satisfactory as he might be, his own mind being apparently tainted in some slight degree with German phraseology and metaphysics. But I shall not trespass further upon ground which lies beyond the limits which it is my present intention to occupy, nor detain the reader from the real subject before us.

Now in order to perceive the possible application of mathematical or demonstrative reasoning to metaphysical subjects, the first requisite is to understand exactly the nature of that reasoning. In order to do this I observe, then,—

First, That it is the important characteristic of mathematical reasoning to proceed from definitions. It sets out from these as precise data, to which appeal is made in every step of the demonstrative process. If these definitions be not clearly understood, if they be not fully granted and well laid up in the mind, in vain does the student or pupil attempt to proceed. The foundations

of the science will then be broken up, the field of mathematical reasoning will then be closed, for these are the gate of entrance.

It affects not the truth of the above position to inquire and settle whether the definitions are hypotheses or facts; whether they be explanations of terms, abstractions of the mind, inductions from observation, or assumptions which have no foundation in the nature of things; nor to inquire whether the definitions of any particular treatise or mathematician, from Euclid and his numerous editors downwards to Newton and his successors, are in every respect the best possible, such as suit best the subsequent course of reasoning, are most easily admitted by the student, or bring most clearly before the mind the principle necessary for future guidance; nor to inquire how far the postulates and axioms partake of the nature of definitions, may be resolved into them, or may be dispensed with altogether, without injury to the study. These may be proper subjects for the metaphysician or logician. They may, to a certain degree, call for the

early attention of the mathematical student; but the tutor who should begin with telling his pupil all that has been said or might be said about the definitions, postulates, and axioms, would probably never get him over the asses' bridge. Suffice it, as a matter of fact, that when you open any elementary treatise on mathematics, Euclid, the conic sections, plane and spherical trigonometry, and even books of arithmetic and algebra, the first objects of careful attention to the student are definitions. These are the foun dations of his science, the elements of his reasoning. To these he must adhere; and if there be anything inconsistent with them, confusion ensues, demonstration ceases.

"It is in this last circumstance (I mean the peculiarity of reasoning from definitions) that the true theory of mathematical reasoning is to be found," says Dugald Stewart in his chapter on Mathematical Reasoning, chap. ii. sect. 3, of his second volume of the Philosophy of the Human Mind, one of the best portions of his writings, yet tinctured deeply with his peculiar faults. This is the

point upon which he rests; and the writers to whom he refers, and with quotations from whom he is so fond of nibbling, will be found substantially to agree with him.

The following passage from Hobbes' Leviathan is also apt to my purpose. "To the priviledge of absurdity, no living creature is subject, but man onely. And of men, those are of all most subject to it, that professe philosophy. For it is most true that Cicero saith of them somewhere; that there can be nothing so absurd, but may be found in the books of philosophers. And the reason is manifest. For there is not one of them that begins his ratiocination from the *definitions*, or explications of the names they are to use; which is a method they are to use onely in geometry; whose conclusions have THEREBY been made indisputable."—Part i. chap. v.

The importance of the definitions admits of the following familiar illustrations. It happened to me to commence the study of Euclid with a youth who stumbled at the first definition,—"A point is that which hath

no parts, or which hath no magnitude." "Then," said he, "it is a nonentity,—it is nothing." He could not or he would not admit such an abstraction; and he began to puzzle himself about the infinite divisibility of matter, the nature and extension of ultimate atoms, the impossibility of finding a given place for that which had no parts, and so on. He was not content to take this or any other definition as a matter of course, and wait to see how far the mathematician would be consistent with himself in his subsequent reasoning. He was determined to weigh and settle the justness of every definition in his own mind before he would proceed further; in short, he would concede nothing and dispute everything. Consequently he never took kindly to mathematical studies; and perhaps to this hour he looks upon mathematics as a multitude of words about nonentities, or things which have no real existence, and consequently no practical value. I by no means imply that in mathematics the student is to begin with submitting to authority, and not to think about the

meaning of the language he uses; nor that the tutor should not be prepared to defend his own preliminary statements. Of this, perhaps, more hereafter. It is sufficient to add at present, that Mr. De Morgan, in his work on mathematical studies, published by the Society for the Diffusion of Useful Knowledge, ranks definition first among the characteristics, and, I may say, as at the foundation, of mathematical reasoning.

Secondly, It may appear to many superfluous, but it is important to observe that the definitions on which mathematical reasoning depends are definitions properly so called; that is, they are explanations of terms —determinations of that sense in which the words employed as signs and instruments of thought are to be taken, used, and understood.

Every one who reads over the definitions of Euclid must, I should think, immediately assent to this. One definition may be better than another of a line, or a straight line, of a circle, or of parallel lines; but its superiority can only consist in fixing

more clearly that sense of the word about to be used, or that quality in the mind's conception of the thing signified, (more simply, the signification,) which alone is to be present to the mind in its subsequent application of the term.

The definitions of geometry concern, it is obvious, the meaning of the terms point, line, straight line, superficies, angle, triangle, circle, and so on, so far as they are definable. And the student would have little occasion to pore over these definitions if all the terms were previously familiar to him, and all had that fixed and clear meaning in his mind; that is, were the signs of those certain ideas of figure for which they stand in the mind of the geometrical reasoner.

If Mr. Dugald Stewart had kept Euclid open before him when speaking of the definitions, I can scarcely imagine he would ever have called them hypotheses; and if he had not called them hypotheses, he would not have maintained that it is the peculiarity of mathematical reasoning to employ hypo-

theses instead of facts as the data on which we proceed. See Stewart's Philosophy, vol. ii. pp. 158, 160.

Take for example the ninth and eleventh definitions. "A plane rectilineal angle is the inclination of two straight lines to one another, which meet together but are not in the same straight line." "An obtuse angle is that which is greater than a right angle." With what propriety can it be said that these are hypotheses? An hypothesis is that mode of accounting for certain appearances which, although probable, remains to be verified by future experiments or observations; or it is the supposed cause of certain effects, whose adequacy or invariable antecedency is assumed until disproved by further investigation. This is the sense in which the term hypothesis is generally employed. In this sense the Ptolemaic and Tycho-Brahic systems of astronomy were hypotheses; in this sense the theory of gravitation, as it first occurred to the mind of Sir Isaac Newton, as accounting for the phases and motions of the heavenly bodies,

was an hypothesis; in this sense the undulatory theory of light is an hypothesis.

Now there is no analogy between this meaning of the term hypothesis and the definitions of geometry, or of any branch of mathematical science. These definitions are not imaginary explanations of given phenomena, nor supposed causes of given effects; they are, as above said, simple explanations of terms, or attempts, by the substitution of other words in place of one general term, to place before the mind, often assisted by diagram or sensible representation to the eye, that object of thought to which the said term is invariably and solely to be applied. "Every general term," says Aristotle, "is the abridgement of a definition."

I observe that Mr. De Morgan, on the Study of Mathematics, p. 70, places reasoning from hypothesis second among the characteristics of geometrical reasoning. "In the statement of every proposition," he says, "certain connexions are supposed to exist, from which it is asserted that certain consequences will follow. Thus, in an isosceles

triangle, the angles at the base are equal, or, if a triangle be isosceles, the angles at the base will be equal. Here the *hypothesis* or *supposition* is, that the triangle has two equal sides; the consequence asserted is, that the angles at the base, or third side, will be equal."

Let us remark, however, that still the hypothesis implies a clear understanding of the words employed, as in the above instance, isosceles and triangle, both of which have been clearly defined and are well understood. Hypothesis here is strictly and merely supposition; a certain figure or relation of lines is supposed or granted to exist, from which certain consequences are deduced. The reasoning would not be valid, or there would be no reasoning at all, if the terms employed did not in the first instance exactly express the thing intended,—the object of thought. Much of common reasoning is reasoning from hypothesis in this sense; that is, it consists in supposing certain relations to exist, and in showing that certain consequences follow.

It was not in this sense that Mr. Dugald Stewart maintained that "in mathematics we employ hypotheses instead of facts" as a general proposition; but rather with intent to show that the whole of mathematics rested upon assumptions, and *therefore* differed from reasonings which turn upon observation, and what he calls facts. He often implies that mathematical reasoning remains good, though there be no such things in reality as points, lines, triangles, circles, and squares in the mathematical sense. But if it be so, even admitting all this, still let us remember that the hypotheses or assumptions, so far as the definitions are included in them, are of a certain kind; namely, that certain words shall invariably be associated with certain meanings or ideas, and no other: for example, that you shall not reason about a triangle as if it could possibly mean a circle, nor about parallel lines as if they were not equidistant at all points.

"It is not on the definition but the conception," Mr. Whewell asserts, "that the properties and demonstrations are built."

But why separate definition and conception? Are they not virtually the same thing? unless by definition we are to understand mere words without signification, little black marks upon a piece of white paper. The definition is of value solely in fixing, and, as it were, embodying the conception. Human beings reason with words, which are the signs or channels of ideas. You can only convey your conception of a straight line or triangle to another mind by a definition or description. It is the object of the definition to single out that quality or property in the mind's conception of the thing which distinguishes it most completely from every other object or thing whatsoever; and which, by being so distinguished, and having such settled property, becomes the subject of reasoning. If it belonged, for instance, to anything else besides an angle to be composed of two straight lines meeting together, but not in the same straight line; in these words we should have no sufficient definition of an angle.

In the Appendix to his work on the Connexion of Number and Magnitude, Mr.

De Morgan makes some admirable and useful observations on the definitions, postulates, and axioms of Euclid; and thus expresses himself:—"Some of the definitions contain assumptions of certain conceptions existing, to which names are to be given; namely, those of a point, a line, the extremities of a line, a straight line, a surface, the extremities of a surface, a plane surface, a plane angle, a plane rectilineal angle; others assume the possibility of certain relations existing, as will appear from the form in which they are put."

He afterwards speaks of these as "indefinable notions," and places the common definitions of them in the light of postulates; thus, "Let it be granted that a point has no parts or magnitude, and that we are concerned with no other property of it, if there be any." Again, he speaks of some of the definitions, those from the eleventh to the fourteenth, and from the nineteenth to the twenty-third, as purely nominal, and therefore needing no remark. From the tenor of his language, which the reader who is not

acquainted with it will do well to consult, it would seem that he considered the geometry of Euclid as resting very much on common notions (κοινη εννοια) which scarcely admit of definition. Nevertheless I do not think that his language countenances any separation between the conception and what is usually considered the definition, but the contrary. The object of all the definitions clearly is to associate a certain term with a particular notion or conception, and thus to fix and limit the meaning of the term. In his paper on the study of mathematics, Mr. De Morgan says, (p. 69.) "This (*i.e.* definition) is merely substituting, instead of a description, the name which it has been agreed to give to whatever bears that description."

In regard to nominal definition, it is to my purpose to quote here what Dr. Whateley says in his Elements of Logic, p. 155, fifth edition; and although I begin the quotation in the middle of a sentence, no alteration is made in its force or meaning:—"all that is requisite for the purposes of reasoning (which is the proper province of logic) is,

that a term shall not be used in different senses; a real definition of anything belongs to the science or system which is employed about that thing. It is to be noted, that in mathematics (and indeed in all strict sciences) the nominal and the real definition exactly coincide; the meaning of the word and the nature of the thing being exactly the same. This holds good also with respect to logical terms, most legal, and many ethical terms."

Upon the whole we conclude that the definitions of geometry settle the meaning of terms.*

Thirdly, These terms are the signs of our ideas of figure and quantity, including in the latter term number and magnitude (both the

* Pascal, in his Reflexions sur la Géometrie en Général, justly observes, however, that many notions are assumed, and terms are used in mathematics which are not defined. "Cette judicieuse science est bien éloignée de definir ces mots primitifs, *espace, temps, mouvement, égalité, majorité, diminution, tout,* et les autres que le monde entend de soi-même." And this must be the case in all reasoning; for, as definition is merely explaining one term by many, it is obvious we might go on defining without end, and not advance a step towards any valuable conclusion.

how many and how great, quantus); which ideas or notions come to us, so to speak, originally from without; *i.e.* they originate in sensible impressions. They are not significant merely of what passes within, or of mental states, like the terms memory, the will, judgement, attention, and desire, unless indeed every sensation, such as of whiteness or blackness, be considered a mental state, and every idea an affection of the mind.

Here, perhaps, I am treading upon the most doubtful, because metaphysical, ground. Right or wrong, however, in what may be said under this head, it will not invalidate what has been said about definition and its object. It appears to me that mathematical reasoning consists in tracing the relations of our ideas of figure and quantity by means of exactly defined symbols, whether words, diagrams, or other symbols, one with another, in respect of agreement or disagreement, equality, or inequality; and these terms and ideas receive clearness and strength by constant application and reference to external things, or sensible impressions; and also by their

observed, clear, uniform, and well-defined relation to each other.

The subject matter of mathematical reasoning may therefore be considered to be real existencies, with as much justice as the subject matter of any other reasoning. For in all reasoning, what has the mind before it but its own abstractions or notions, and terms affixed to those notions? And who can say that circles, angles, squares, lines, have not as much foundation in, and reference to, things as they exist, as white, blue, black, soft, hard, or other qualities of body, solid, liquid, brittle, or elastic; or the abstract ideas of space, time, beauty, honour, virtue, and so on? Our ideas of number and figure are ideas constantly forced upon us by sensible objects, and all that fills this visible diurnal sphere; the terms significant of these ideas are in constant use and application in ordinary life. They are employed by the humblest in station and education with uniformity of meaning, with clearness and accuracy for their purposes. It is true they may not know anything of the rela-

tions and properties of triangles, squares, circles, parallelograms, as traced by the mathematician; but the mathematician's skill and wisdom consist only in having traced and studied these relations by means of his exact definitions, and by his deeper or more frequent meditation on their several connexions and consequences. The ideas or notions of number and figure are common to all minds. Attention and instruction only are necessary to furnish them with the exact definitions and new combinations. In number, it is obvious that the terms or figures are themselves definitions, or their equivalents.

It is because the subject matter of mathematical reasoning consists, in our ideas of figures and magnitudes or quantities, that the reasoning may be carried on by other signs than words, viz. sensible diagrams. The Arabic numerals, and the notations of algebra, are artificial contrivances or abbreviated symbols for tracing the relations of quantity as they are wanted, or as those relations follow from the nature of the contrivances themselves. These diagrams, these

figures and notations, are the signs and instruments of the mathematician's or algebraist's thoughts; and it is because they are always of a clear and certain nature, and bear a uniform, fixed, and definite relation one to another, that the geometrical reasoning, and the arithmetical and algebraic processes are the same to every mind.

Upon this circumstance, namely, the power of fixing the attention and carrying on the reasoning by means or help of sensible diagrams, Locke fastens, as of the first importance, and the great peculiarity in mathematical studies.

"That which has given the advantage to the ideas of quantity, and made them thought more capable of certainty and demonstration, is, first, that they can be set down and represented by visible marks, which have a greater and nearer correspondence with them than any words or sounds whatsoever. Diagrams drawn on paper are copies of the ideas in the mind, and not liable to the uncertainty that words carry in their signification. An angle, circle, or square, drawn in lines,

lies open to the view, and cannot be mistaken: it remains unchangeable, and may at leisure be considered and examined, and the demonstration be revised, and all the parts of it may be gone over more than once without any danger of the least change in the ideas. This cannot be thus done in moral ideas; we have no sensible marks that resemble them, whereby we can set them down."—(Book IV., chap. iii., § 19.)

It matters not that he is the best mathematician, or arithmetician, who needs least the sensible diagram, or the figure on the paper; nor to say, with Mr. Stewart, that the figure on paper cannot pretend to that precise exactness which is the object of our reasoning; that the line we draw will have some breadth, and the circle, however steady the instrument and the hand, may deviate in some point from equidistance. The most skilful reasoners can only have a certain idea of visible figure, and of the relation of its several parts present to their minds, which the less skilful require for facility and permanency of reference on the paper. The

diagram approaches sufficiently to sensible exactness to keep before the mind that quality of the figure which is the sole object of the reasoning; and it is sufficient that the more nearly the specific figure before us approaches to exactness, the more applicable will the reasoning be to that figure,—or, more correctly, it is only in so far as the figure fairly represents the mind's view of its qualities that the reasoning applies to it at all. There is no such mystery in the most obscure of the definitions as to make us deny their reference to a certain specific quality of objects, that is, to real existencies, in the only practical sense of the words. The constant application of mathematical reasonings to the various branches of natural philosophy, and the common use of mathematical terms in the mathematical sense, prove the contrary. We speak, for instance, of the line between one shade of colour and another, and length without breadth is the only object of the mind's contemplation in so speaking. Points and angles are words of perpetual occurrence: the

former in the sense of the commencement or termination of lines, without being any decided parts or given portions of the line; and the latter in the sense of the meeting of two or more lines together, converging or diverging with more or less of rapidity or extension.

It is perhaps of little consequence to determine whence we get the notions or conceptions upon which mathematical reasoning turns, whilst it is certain we have the notions and defined terms appropriate to them, except in so far as it appears that in numerical calculations, and in the geometry of Euclid, there is a certain verification of the reasoning by an appeal to the evidence of the senses. In fact, it is hard to divine whence we get notions of figure or quantity if it be not from the sight and the touch, or from experience,—a word of extensive signification, comprehending all the results of observation and reflection. Those who say we do not get these notions or conceptions from experience, would do well to tell us whence we do get them, or produce the mathematician

upon whom God has not bestowed the five senses with which he has happily blessed the rest of mankind.

I will here venture a remark upon Mr. Whewell's language, in his pamphlet on Mathematical Studies (second edition, p. 32):

"I mentioned it," says Mr. Whewell, "as likely to make the study of mathematics less beneficial as a mental discipline than it might otherwise be if the first principles of our knowledge be represented as borrowed from experience, in such a manner that the whole science becomes empirical only.

"I will not suppose that any person who has paid any attention to mathematics does not see clearly the difference between necessary truths and empirical facts,—between the evidence of the properties of a triangle and that of the general laws of the structure of plants. The peculiar character of mathematical truth is, that it is *necessarily* and *inevitably* true; and one of the most important lessons which we learn from our mathematical studies is a knowledge that there

are such truths, and a familiarity with their form and character.

"This lesson is not only lost, but read backwards, if the student is taught that there is no such difference, and that mathematical truths themselves are learnt by experience. I can hardly suppose that any mathematician would hold such an opinion with regard to geometrical truths, although it has been entertained by metaphysicians of no inconsiderable acuteness, as Hume. We might ask such persons how experience can show, not only that a thing *is*, but that it must be; by what authority he, the mere recorder of the actual occurrences of the past, pronounces upon all possible cases, though as yet to be tried hereafter only, or probably never. Or, descending to particulars, when it is maintained that it is from experience alone that we know that two straight lines cannot enclose space, we ask, who ever made the trial, and how? And we request to be informed in what way he ascertained that the lines with which he

made his experiment were accurately straight. The fallacy is in this case, I conceive, too palpable to require to be dwelt upon."

A meaning of the word empirical has crept into our language lately, in consequence of the freedom with which some philosophers treat the king's English, and I fear also from the bad translation of some German writings, which it was not wont to have, as if it were simply equivalent to experimental; whereby we are threatened with the loss of a good word for a very important idea, namely, that of quackery, or the observance of rules drawn from a narrow experience, in neglect or defiance of a large and true experience. In eight instances of the use of the word by our best old English authors, which Johnson gives, it is invariably associated with this latter meaning. No fact, which is a fact, can deserve the epithet empirical.

Mr. Whewell would have done well, if we do not get our knowledge of the first principles, or, as he better expresses it, the fundamental conceptions of mathema-

tical science from experience, to inform us whence or how we do get them. As he has not supplied that information, his reader may be apt to pause; and if he be a friend of that wise and cautious old gentleman hight Experience, he will not easily allow the laugh to be turned against him. Just so Mr. Dugald Stewart, in his remarks upon demonstrative reasoning, says, "It is by no means sufficient to account for the essential distinction which every person must perceive between the irresistible cogency of a mathematical demonstration and that of any other process of reasoning:" "that, in mathematics, there is no such thing as an ambiguous word." But Mr. Stewart does not help his reader to account for it in any other way. Thus he first plunders him of an all-sufficient principle, and then leaves him in the dark; nay, he lays it down as his own principle, that it is the peculiarity of mathematics to reason from definition, as if keeping to himself what he would not allow to another.*

* See on this the passage of Du Hamel's, quoted by the Edinburgh reviewer, p. 427.

It appears to me very reasonable to ask, "What but experience can show, not only that a thing is, but that it *must be?*"—a very general and perhaps useless proposition. —*Experientia docet*; and let every man be careful how he limits the extent and value of her lessons. As we did not make our own senses, nor the external world, we are supplied by the constitution of our frame with certain conceptions which are natural to, and inseparable from, that frame. We make words or signs for our conceptions, and by use the words become indissolubly associated with those conceptions; and so long as we make those signs or words stand for those certain conceptions, so long, in fact, as being *signs* they have signification, we act the part of rational and consistent beings. If a plain man be asked how he knows that two straight lines cannot enclose space, he may, in his turn, ask the questioner whether he ever knew it otherwise; and so may force him to own that constant experience taught him that truth; that nature had furnished him with the notions of a line and

of straightness, and the words belonging to those notions; that his mathematical studies had built upon that experience; and that, in regard to ascertaining that any given lines before him were accurately straight, it was clear that the straighter they were, in any conceivable meaning of the term straight, the less likely they were to enclose a space. If I were asked how I know that in any right-angled triangle the square which is described upon the side subtending the right angle is equal to the squares described upon the sides which contain the right angle, my first answer might be, that I knew it by studying the 47th of the first book of Euclid. But the study of Euclid forms a small part of my experience, which includes all my observations and reflections upon the contents of Euclid, and all the conceptions gathered from the study of the relations traced and traceable between the various figures therein the subject of meditation. In short, experience, like nature, is a word of such very comprehensive import, as containing within itself so completely the sources of knowledge and instruction, that

whatever does not fall within the boundaries of that wide domain, can be nothing short of immediate inspiration.* If a friend asks me to *show him* that the thing must be so, or in other words to furnish him with Euclid's proof, or a mathematical proof of the truth of that 47th proposition, that is another question; and then I should recal the steps of the demonstration; and what demonstration is, is the matter into which we are now inquiring.

There is a well-known work by Dr. Beddoes, on the nature of Demonstrative Evidence, which contains many useful observations on the connexion between language and thought, in which he endeavours to show that Euclid's reasoning begins from experiment and proceeds by experiments. There is an awkward-

* Of course it would be absurd to contend that the truths and demonstrations of geometry are lessons of *mere experience* in a sense strictly analogous to that in which we apply the term to the observations and details of our ordinary daily existence and sensation. We are discussing solely the origin of the fundamental conceptions on which mathematical reasoning rests—the data from which it starts.

ness in the phrase *mental* experiments, which the Doctor uses, and which might have been avoided by a different mode of stating his argument or view; and which seems to be this,—that the fundamental notions or conceptions from which mathematical reasoning starts, and to which it appeals, are as much the result of experience, and rest as much upon the evidence of the senses, and the natural meaning of our own words, in connexion with that evidence, as the fundamentals of any physical science whatsoever; and he instances particularly the axioms, as they are called, that "two straight lines cannot enclose a space," and "the whole is greater than its part."

In a review of a treatise of Leslie's, on mathematics, attributed to Professor Playfair, in the twentieth volume of the Edinburgh Review, there are some remarks upon this work of Dr. Beddoes, which, coming from Professor Playfair, are entitled to particular consideration. Playfair suggests that Beddoes was no great mathematician. But with submission, this is no

answer to Beddoes' argument, and rather too near an approach to the common tactics of controversial writing, in which the reader's attention is diverted from the question, and the pursuit of truth, by some insinuation against the character or abilities of an adversary. Playfair tells us that "geometrical reasoning is a process purely intellectual, and resting ultimately on *truths* which the mind *intuitively perceives*." Are we, then, to rest here without going further,—without venturing to ask what are "truths intuitively perceived"? In what sense this is true, the present observations are meant to illustrate, and, if I am not very much deceived, will sufficiently, or in a great measure, help the reader to understand. Meantime I beg to call his attention to a remarkable and just sentence of Hartley's, in his invaluable and profound chapters on "Words, and the Ideas associated with them, and on Propositions and the Nature of Assent." "Rational assent to any proposition may be defined a readiness to affirm it to be true, proceeding from a close association of the ideas sug-

gested by the proposition, with the idea or internal feeling belonging to the word truth, or of the terms of the proposition with the word truth ;" and then follow some observations on geometrical and mathematical reasoning, which are as clear, beautiful, and unanswerable, as any observations upon abstract truths within the circle of human science and philosophy.

But, fourthly, whencesoever we get the notions or conceptions with which we are concerned in mathematical reasoning, I think it must be admitted, that habit, *i. e.* the constant recurrence of the same simple ideas of number and figure, and the constant association of the same terms with the same ideas, has much to do with that feeling of certainty and satisfaction, that readiness and confidence of assent, which we recognise in connexion with the processes of arithmetic, algebra, and geometry.

How much there is in habit may be easily and irresistibly shown. Thus we say that 2 and 3 make 5, and the three angles of a triangle are equal to two right angles;

and we feel the truth as we pronounce the words. But if we take higher numbers, and more advanced propositions,—if we say that nine thousand six hundred and seventy-three (9673) times seventy-three thousand six hundred and nine (73,609) make 712,019,857, or upwards of seven hundred and twelve millions; or if we take some of the propositions relating to proportion in the fifth book, or go on to the more abstruse calculations in algebra, trigonometry, and fluxions, will our assent be so ready? Who will assert it? And why?—because we are not in the habit of attending to high numbers and advanced propositions. Doubt, ignorance, and difficulty attach themselves to our terms. He who has just risen from calculations, or the study of mathematics, will feel a confidence in terms and propositions which others do not. A ready accountant casts up with a glance or two a long column of accounts; he perceives the relation of each item to the whole amount in a space of time that appears incredibly short to one wholly unaccustomed to such work. Those who are in the habit of

estimating the number of persons in a crowded room or assembly, can tell by looking at the mass, with reference to the space occupied, how many may be present with much more correctness than another who should try for the first time to count the heads. So the bare statement of a proposition, and a glance at the diagram, will enable the quick mathematician to understand the whole demonstration, and to repeat the various steps of the process faithfully to another; while he who is slow at combining the ideas of figure, notwithstanding ever so careful reading of the proof, will be still at a loss to perceive its cogency; and will pass from the words to the figure, and the figure to the words, without being a whit the wiser, or having any distinct idea of what he is about, or where he is, present to the mind. The elaborate paper of Sir W. Hamilton, of Dublin, to the Royal Society*, appears a chaos of warring elements, a mere jumble of letters and figures to the tyro

* "On a General Method in Dynamics."—Phil. Trans. 1834, Pt. ii., p. 247.

in algebraic studies; "monstrum horrendum inform' ingens cui lumen ademtum;" but to that of the learned reader, and to his own eye, it appears as the harmonious and beautiful arrangement of simple elements, each having its due place and force, combining to one noble, important, and useful result.

Further, in geometrical and mathematical reasoning the premises are few; the terms employed are few; and the mind is only engaged in tracing the relations of a few distinct simple ideas, which are fixed by sensible impressions. The whole vocabulary of Euclid may be comprised in a couple of pages. Each book turns upon a few definitions. The whole volume is filled with repetitions of the same terms, with appeals to the same brief premises; attention is more or less frequently recalled to each proposition as it passes in review, and which ranks, when proved, among the foregone premises. The notations of algebra are comparatively few; the letters which stand for unknown quantities derive their meaning solely from connexion with, and relation to, the known quantities, at least in their first

use; and at last from their relation to each other, in consequence of an extended meaning in the symbols, with which meaning, by habitual contemplation, the mind becomes familiar. Among the figures of arithmetic there are but nine units; after ten you begin with new relations of the first nine; hundreds are combinations of tens, thousands of hundreds, and so on. And with regard to the higher numbers, we can always make clear their value to the senses; for though we could form not the least notion how many men there might be in a field of battle, or how many grains of corn in a sack, by looking at them in the mass, yet divide them into companies of thousands, of hundreds, and tens, and by this arrangement the mind gains a clear and practical sense of the number. It is doubtless by understanding the number and character, and the due arrangement of his forces, that a commander-in-chief is enabled to dispose of them to the best advantage, and form the order of battle.

Our ideas of number and figure are what Locke calls "distinct simple modes;" and however varied in combination or relations,

the same signs or terms are invariably connected with the same conformations of figure, and the same relations of number. Put down a three-sided figure in lines, or any four or more of the Arabic numerals in a line, as 4565, and every human being using the English language would express the relation in the same terms,—would pronounce the one a triangle, and read the other four thousand five hundred and sixty-five.

"The idea of *two* is as distinct from that of one," says Locke, B. II., chap. xiii., "as blueness from heat, or either of them from any number; and yet it is made up only of that simple idea of an unit repeated; and repetitions of this kind joined together, make those distinct simple modes of a dozen, a gross, a million."

Thus also he speaks concerning figure, § 6: "The mind having a power to repeat the idea of any length directly stretched out, and join it to another in the same direction, which is to double the length of that straight line, or else join another with what inclination it thinks fit, and so make what sort of

angle it pleases; and being able also to shorten any line it imagines by taking from it one-half, or one-fourth, or what part it pleases, without being able to come to an end of any such division, it can make an angle of any bigness; so also the lines that are its sides of any length it pleases, which joining again to other lines of different lengths, and at different angles, till it has wholly enclosed any space; it is evident that it can multiply figures, both in their shape and capacity, *in infinitum;* all which are but so many different simple modes of space."

There does not appear any advantage, but the contrary, in the use of the term "*mode*," and alternating it with "*idea*," as Locke does in this and in other parts of his Essay; but whether ideas or modes, it is evident they are simple, because they do not admit of being resolved into other ideas or notions still simpler, but result at once from uniformity in the structure and impressions of the senses, which uniformity lays the foundation for language and reasoning.

The simplicity and uniformity of the sen-

sible impressions of space, or figure and number, and the comparative fewness of the terms or symbols in use in mathematical reasoning, constantly associated with the same impressions,—terms or symbols which are in fact human contrivances for conveying those impressions from one mind to another,—these things are to be borne in mind, and duly weighed, in estimating the nature of demonstrative evidence. Nor let any man despise mathematical studies, or think them a mere ringing of changes upon the same set of bells, because the terms employed are few, and the original simple ideas few; otherwise, let him despise the English language, or language in general, because there are only twenty-six letters in the alphabet. For what endless varieties of thought,—what worlds of wisdom,—what vast structures of science, are these twenty-six letters, all-sufficient! And what would human life be without them?

But we have not yet analysed the nature of mathematical reasoning. We have said that mathematical reasoning sets out from

definitions; that these definitions settle the meaning of terms; that these terms are the signs of our ideas of figure and quantity, of numbers and magnitudes; that these ideas are among the simplest, clearest, with which our minds and senses are conversant; that the terms in use, and the simple ideas to which they are uniformly appropriated, are comparatively few; that the constancy of connexion between the terms and ideas, that is, habit, has much to do with that feeling of assent and conviction to which the reasoning gives rise, by which the processes are accompanied, as any one must perceive who begins to instruct children in arithmetic or in geometry.

But further, fifthly, and lastly, the demonstrative quality of mathematical reasoning consists essentially in this,—the perception of the agreement or disagreement of certain ideas and certain terms with other intermediate ideas and terms, which are used as a measure or test of truth, such ideas and terms having been previously selected by the mind for a measure or test; in other words,

a means or standard of comparison. *To demonstrate is to show that a certain proposition not granted to be true is true by virtue of some premise previously admitted or assumed as a criterion of truth, or by virtue of some other truth previously demonstrated.** In other words, to demonstrate is to discover and trace further and new relations amongst our ideas by comparing them one with another; which new relations will be of determinate and constant character, in proportion as the intermediate ideas which are used as a means of comparison, as a measure, are themselves of distinct and constant character and value. Or again, it is to show that ideas not clearly perceived to harmonize or agree, do harmonize, by comparison, with other ideas whose agreement is clearly perceived. Thus in the first proposition of Euclid, the sides of a given triangle are proved to be equal when they are all shown to belong to the class of lines which radiate

* The reader may consult the papers on mathematics, by Mr. De Morgan, who gives this just account of demonstration.

from the centre to the circumference of the same circle, or of equal circles; of which class of lines equality is previously premised in the fifteenth definition.

This essential quality of demonstrative reasoning is thus distinctly laid down by the great master Locke, book iv., c. ii. :—

"The next degree of knowledge is where the mind perceives the agreement or disagreement of any ideas, but not immediately. Though wherever the mind perceives the agreement or disagreement of any of its ideas there be certain knowledge, yet it does not always happen that the mind sees that agreement or disagreement which there is between them, even where it is discoverable; and in that case remains in ignorance, at most gets no further than a probable conjecture. The reason why the mind cannot always perceive presently the agreement or disagreement between two ideas, is because those ideas, concerning whose agreement or disagreement the inquiry is made, cannot by the mind be so put together as to shew it. In this case, then, when the mind cannot so

bring its ideas together, as by their immediate comparison, and, as it were, juxtaposition and application one to another, to perceive their agreement or disagreement, it is fain, by the intervention of other ideas, (one or more as it happens,) to discover the agreement or disagreement which it searches; and this is what we call *reasoning*."

Again, § 3 :—

"Those intervening ideas which serve to show the agreement of any two others are called proofs; and where the agreement and disagreement is by this means plainly and clearly perceived, it is called demonstration."

Again, § 7 :—

"In every step reason makes in demonstrative knowledge, there is an intuitive knowledge of that agreement or disagreement it seeks with the next intermediate idea, which it uses as a proof; for if it were not so, that yet would need a proof; since without the perception of such agreement or disagreement there is no knowledge. If it be perceived by itself, it is intuitive knowledge; if it cannot be perceived by itself, there is

need of some intervening idea, as a *common measure* to show their agreement or disagreement. * * * * So that to make a thing a demonstration, it is necessary to perceive the immediate agreement of the intervening ideas, whereby the agreement or disagreement of the two ideas under examination (whereof the one is always the first and the other the last in the account) is found."

It is not without reason that Locke dwells upon this; and he repeats himself in ch. xv. of the fourth book on Probability, which the reader may consult.

In this analysis of demonstrative or mathematical reasoning, it is finally to be observed that the definitions are used as the primary common measures or tests; they are the original ideas or settled notions by means of which the relations of other ideas one with another are traced, and the agreement or disagreement ascertained and settled, and by which the new relations, so ascertained, become themselves of determinate and constant character. Each book begins with its necessary definitions; and each proposition,

when settled, becomes itself a premise or test, by help of which further relations are traced, and new agreements or disagreements ascertained and fixed. The mind is continually reverting to its original simple notions, builds carefully upon them, and not only has a power to retrace, but is very frequently employed in carefully retracing every step of its progress. Thus we return to the point from which we set out, that definition is the basis of mathematical reasoning, and gives it its peculiarly fixed, clear, and certain character.

The reader who may doubt whether this be a correct or perfect analysis of mathematical or demonstrative reasoning, is requested, by a careful examination of mathematical works, to supply the deficiency. Let him apply it to the most simple or the most abstruse propositions and demonstrations, and say what essential quality of such reasoning has been omitted.

SECTION II.

Having now analysed with all the completeness in our power, the nature of demonstrative reasoning, we are prepared for the inquiry, whether it differs from other reasoning, or reasoning in general, in any respects or particulars whatsoever. And, if it do not so differ, we are then prepared for the important inquiry, how the cogency and certainty of mathematical science can be applied to and obtained in moral, political, metaphysical, and religious subjects.

Now the tendency and almost the object of Mr. Dugald Stewart's chapters on mathematical demonstration, and on the Aristotelian logic, is to draw a broad line of distinction between mathematical reasoning, mathematical evidence, and other kinds of reasoning, other kinds of evidence. "Mathematical definitions," he says, (vol. ii., p. 156, 4to,) "are of a nature essentially different from the definitions employed in any

of the other sciences." Again, p. 157, he speaks of "the essential distinction which every person must perceive between the irresistible cogency of a mathematical demonstration and that of any other process of reasoning." He repeats this idea in various places. I need only refer to p. 203, where he says, "If the account which has been given of the nature of demonstrative evidence be admitted, the province over which it extends must be limited almost entirely to the objects of pure mathematics."

But what says Dr. Whately in his Elements of Logic?

"One of the chief impediments to the attainment of a just view of the nature and objects of logic, is the not fully understanding, or not sufficiently keeping in mind, *the sameness of the reasoning process in all cases.* If, as the ordinary mode of speaking would seem to indicate, mathematical reasoning, and theological, and metaphysical, and political, &c., were essentially different from each other, *i. e.* different kinds of reasoning, it would follow that, supposing there could be at all any such science as we have described logic, there must be so many different species, or at least different branches of logic. And such

is perhaps the most prevailing notion."—3rd. ed. p. 23.

Again, p. 25, he says,—

"Supposing it to have been perceived that the operation of reasoning is *in all cases the same,* the analysis of that operation could not fail to strike the mind as an interesting matter of inquiry."

And thus, p. 50:—

"Whatever the subject matter of an argument may be, the reasoning itself, considered by itself, is in every case the same process; and if the writers against logic had kept this in mind, they would have been cautious of expressing their contempt of what they call syllogistic reasoning, which is in truth all reasoning."

Let us contrast with this,—for there is nothing more instructive than bringing into juxtaposition the different aspects in which these recondite matters are presented to our attention,—let us contrast with this the barely intelligible language of the Edinburgh reviewer, p. 413.

"Now as all matter is either necessary or contingent, (a distinction which may be here roughly assumed to coincide with mathematical or non-

mathematical,) we have thus, besides a theoretic or general logic, two practical or special logics in their highest universality or contrast.

"THEORETICAL LOGIC.

Practical logic, as specially applied to necessary matters = mathematical reasoning.	Practical logic, as specially applied to contingent matter = philosophy and general reasoning."

He says, p. 422,—

"How opposite are the habitudes of mind which the study of the mathematical and the study of the philosophical sciences require and cultivate, has attracted the attention of observers from the most ancient times. The principle of this contrast lies in their different objects, in their different ends, and in the different modes of considering their objects."

He speaks also of mathematics as "determining dissimilar developments of thought from other sciences," as "not developing the higher faculties," as "dependant on the lower imagination."

Again, p. 422 :—

"Mathematics, departing from certain original hypotheses, and these hypotheses exclusively determining every movement of their procedure, and

the images or vicarious symbols, about which they are conversant, being clear and simple, the deductions of these sciences are apodictic or demonstrative; that is, the possibility of the contrary is at every step seen to be excluded in the very comprehension of the terms. On the other hand, in philosophy, (with the exception of the science of logic,) and in our reasonings in general, such demonstrative certainty is rarely to be attained; *probable certainty, i. e.* where we are never conscious of the impossibility of the contrary, is all that can be compassed; and this also not being internally evolved from any fundamental data, must be sought for, collected, and applied from without."

"From this general contrast it will be seen how an excessive study of the mathematical sciences, not only does not prepare, but absolutely incapacitates the mind for those intellectual energies which philosophy and life require. We are thus disqualified for observation either internal or external, for abstraction and generalization, and for common reasoning."

Now common reasoning we conceive to be very bad reasoning; such reasoning as fails to satisfy the man who is seeking for clear and exact views, who fears to be misled by words, and who remembers that fine phraseology teaches nothing. It may be observed

here, that whatever force or justness there is in the reviewer's general course of observation, it all lies in the word "excessive"—"an *excessive* study of the mathematical sciences." And it is perfectly obvious that he who is conversant only with mathematical notions and mathematical processes, may be ignorant of many other objects of human attention, which come nearer home to the business and bosoms, the pleasures and pains, of mankind at large. He who is always dwelling in circles and squares, ellipses and parabolas, differentials and integrals, may have a proportionally confined range of thought. He will not understand the feelings and thoughts of other men; and he may fancy, from the habitual association of his ideas, or from his determining everything in the same way, that he can ascertain the precise quantity of enjoyment which a company of aldermen derive from eating and drinking, by means of the differential or integral calculus, and determine the relative merits of Homer and Virgil by the rule and compasses. But what then? Shall mathematical studies not be

valued as an essential part of the training of the youthful mind? Is Mr. Whewell's sentiment invalidated, that they are the best practical exemplification and exercise of logic? If there be one mode of studying mathematics better than another, shall not a mathematical professor discuss this question, and endeavour to settle which is best? How many sciences are there which require for their pursuit, comprehension, and enjoyment, a thorough knowledge of the higher branches of mathematics, such as astronomy, optics, dynamics, and all those which go under the name of the mixed sciences. Who would undervalue the highest mathematical attainment when applied to these branches of science; and not rather regret, when he sees the mathematician soaring in the clouds and lost in the dim distance of algebraic formulæ, his inability to follow? "Non omnes possumus omnia." But we can all enjoy and apply those practical and simple conclusions, for the establishment of which the most profound mathematical investigations are oftentimes necessary.

If the question be, What degree of time and attention should be given up to mathematical studies in a thoroughly comprehensive course of academic education? or, How far exclusive encouragement should be given to high mathematical attainment in an university? (which the reviewer has in part raised and discussed,) this may be settled without depreciating the importance and value of mathematics for the discipline of the youthful mind. You have then to take into account the great and general purposes of education, the whole constitution of the human mind, the condition, and wants of society at large, the fitness of an individual for the particular station which he is designed to occupy, and the kind of knowledge which his meditated profession may require.

It is curious to contrast the reviewer's statement of the injurious influence of mathematical science in disqualifying for observation, either internal or external, for abstraction and generalization, with the intellectual character of Sir Isaac Newton

drawn by Sir John Herschel in his Treatise on the Study of Natural Philosophy, p. 271.

"His wonderful combination of mathematical skill with physical research enabled him to invent, at pleasure, new and unheard-of methods of investigating the effects of those causes which his clear and penetrating mind detected in operation. Whatever department of science he touched, he may be said to have formed afresh. Ascending by a series of close-compacted inductive arguments to the highest axioms of dynamical science, he succeeded in applying them to the complete explanation of all the great astronomical phenomena, and many of the minuter and more enigmatical ones. In doing this, he had every thing to create; the mathematics of his age proved totally inadequate to grapple with the numerous difficulties which were to be overcome. * * * Of the optical discoveries of Newton we have already spoken; and if the magnitude of the objects of his astronomical discoveries excite our admiration of the mental powers which could so familiarly grasp them, the minuteness of the researches into which he there set the first example of entering, is no less calculated to produce a corresponding impression. Whichever way we turn our view, we find ourselves compelled to bow before his genius, and to assign to the name of *Newton* a place in our veneration which belongs to no other in the annals of science. His era marks the accomplished

maturity of the human reason as applied to such objects. Every thing which went before might be more properly compared to the first imperfect attempts of childhood, or the essays of inexpert though promising adolescence. Whatever has been since performed, however great in itself, and worthy of so splendid and auspicious a beginning, has never, in point of intellectual effort, surpassed that astonishing one which produced the Principia."

I refer to this treatise with a strong feeling of interest, because it is evident from the observations on nomenclature, and on science generally, that Herschel's clear English mind duly estimates the importance of settled terms with settled meanings; and while he dwells on the necessity of having exact and uniform standards of measurement and value, his reader is set upon the inquiry into the nature and purposes of measures or tests. He who can perceive the importance of a proper use of words in physical science, must feel that importance also in metaphysical. Without it, in fact, we can have nothing worthy of the name of science. Sir John Herschel would probably

smile at the idea of mathematical science disqualifying for generalization and abstraction or any useful exercise of mind.

I have indulged in these references to Dugald Stewart, Dr. Whately, the Edinburgh reviewer, and Sir John Herschel, with a view to place before the reader in an easy manner the different lights in which the same objects, or objects closely allied in nature and character, are presented to our attention, and the necessity of close and cautious investigation.

Now bearing in mind the foregone analysis of geometrical or demonstrative reasoning, in order to perceive its connexion with logic, it is necessary to understand what logic is. Is Dr. Whately right or wrong when he says the reasoning process is the same in all cases? If he is right, of course it follows that mathematical or geometrical reasoning is but one illustration or practical application of logic.

I am unable to attach any other consistent meaning to the term logic, than that it is another word—the Greek word—for reasoning.

As a science it investigates the principles of reasoning, or analyses and determines the process of the mind in reasoning; as an art it is the practical application or exemplification of the rules so deduced. On this point nothing can be clearer and more satisfactory than Dr. Whately's observations in his preface and throughout his treatise.

Yet notwithstanding this clearness, and notwithstanding Dr. Whately's correction of the error of Watts in considering logic as "the right use of reason," "a method of invigorating and properly directing all the powers of the mind," a writer on logic in the edition of the Encyclopædia Britannica now in the course of publication, says, "Logic may be defined as *the science of the laws of thought considered as thought*. This is the central notion towards which the various views of the science, from Aristotle downwards, gravitate; it is the one definition in which others, apparently the most opposite, find their complement and reconciliation." Then, by way of elucidating this definition, the writer (whom from his use of the term laws, and the

epithets, contingent, necessary, universal, dirigible, and so on, I could suspect to be the Edinburgh reviewer already alluded to) proceeds to tell us, first, that logic is conversant about thought; in the second place, about thought considered as thought; and in the third place, it is the science of the laws of thought, because it is conversant about the universal and necessary in thought.

These are the remarks of a writer who comments on the erroneous definition of logic, in an article which the editor of the Encyclopædia has reprinted; an article which tells us that "Logic is the art of properly conducting reason in the knowledge of things, whether for instructing ourselves or others; or it may be defined the science of human thought, inasmuch as *it traces the progress of knowledge*; and that its business is to evolve the laws of human thought, and the proper manner of conducting the reason, in order to the attainment of truth and knowledge." And while the writer comments on this article, he further tells us that from Aristotle downwards the purity of the science

has been contaminated by foreign infusions. He speaks of Dr. Whately's Elements as vague and vacillating in its views, its doctrines neither being developed from the primary laws of thought, nor combined together as the essential parts of one necessary whole. In short, being desirous to make something more of a subject than has ever yet been made of it, and to see further into things than any one else has seen, he plunges into darkness and a wood of words,

. . . . . "hunc tegit omnis
Lucus, et obscuris claudunt convallibus umbræ,"

or, like many of his brethren, he is so blinded by the mists of his own land that he cannot enjoy the cheerful sun and daylight loved by the children of the south; and when he is pleased to consider thought as thought, he forgets that no one in his senses was likely to mistake it for "cakes and ale." Indeed, but for the eminence to which the Encyclopædia Britannica aspires, and is in part deservedly raised as an authority in the sciences, he might be benevolently left to the condition and neglect in which the New

Poor-Law leaves those who will not help themselves when they can.

Seriously, however, when we talk of the science of the laws of thought, do we know what we are talking about? With confidence, I answer No. The whole subject of metaphysics, the whole state of our knowledge and language on the nature, qualities, powers, and affections of the human understanding, as may be inferred from the article metaphysics in this very Encyclopædia, is such, that to talk of the laws of thought, to speak of primary laws, which implies secondary, and universal and necessary, which implies particular and contingent, is to talk of nobody knows what. What has logic to do with the laws by which thoughts come and go in the mind of a child or of a maniac? for I suppose a maniac has thoughts, and, if so, is subject to laws of thought. True enough, all logic supposes a thinking mind; but so does every other science; so do carpentry and masonry; and wherever we have thinking minds, there we have minds subject no doubt to what we are pleased to call laws.

But to set the mind hunting after the general laws of thought, under pretence of studying logic, is to entrap the student into unlooked-for difficulties, to leave Aristotle utterly in the lurch, to give us our labour for our pains, and to bring us, after a fatiguing hunt, like Spenser's good knight, only to the cave of despair. If logic be the science of the laws of thought, what is the province of mental philosophy? I do not question that the one touches closely the province of the other, but science used formerly to consist in nicely distinguishing rather than confounding the things that differ, howsoever minute that difference. At the risk of appearing merely to reprint what my reader may find elsewhere, but what cannot be too strongly impressed upon the mind, I must use the words of Dr. Whately, and say that "the attempt to comprehend so wide a field is no extension of science, but a mere verbal generalization, which leads only to vague and barren declamation. In every pursuit, the more precise and definite our

object, the more likely we are to attain some valuable result."

Without further discussion, I must assume that logic is but another word for reasoning; and the object of it as a science is to ascertain the process of the mind, to which we specially give that name. Now we have analysed the nature of mathematical reasoning, or, in other words, we have examined the process of the mind in that reasoning. Can we, then, abstract what is peculiar to the mathematics, and talk of reasoning in general without regard to any particulars? May we not ask what is meant by reasoning as a term standing alone? Is there one determinate process of mind to which the term reasoning is peculiarly and alone appropriate?

Nature and the senses give me the idea of a man and of a horse. I suppose the body and legs of a horse joined to the breast and head of a man, and call that supposition or conception by the name of a centaur. Doing this, would you say I reason? No, I only

imagine, and give a name to the object of my imagination, which are indeed important elements of the reasoning powers. But when I say all animals have feeling, no vegetable has feeling, therefore no vegetable is an animal; you would say I reason, although from the very obviousness of the words, and from their arrangement, and the smallness of the effort of which we are conscious in following that arrangement, the portion of reason concerned, if we could divide reason into measureable portions, is almost too insignificant to be worthy of the name. But if this be reasoning, what have we?

First of all, words, or audible sounds associated with many sensible impressions or objects—animals.

Secondly, These objects classified, and viewed in a common relation or under the affirmation—having feelings.

And thirdly, Other objects, viewed under a different relation, having no feeling, therefore excluded from this class, no vegetables animals, or vegetables no animals.

In this who can detect any thing but the

results or lessons of human experience or registered observation, classified, and clothed in appropriate language, — that which is affirmed of one being denied of the other class, — language being to us the means and very element of thought, at least of thought conveyed from one mind to another ? and hence the beauty of the Greek word λογος, which is at once verbum and ratio—the audible sound and mental apprehension.

I prefer, however, taking a work of authority like Dr. Whately's as a guide for the course of thought which it appears most useful and important to pursue. In analysing the operation of the mind in reasoning, Dr. Whately says, "It will be found that every conclusion is deduced in reality from two other propositions, thence called premises." He contends there must be two propositions, and says, (section third,) of a valid argument, "It is impossible for any one who admits both premises to avoid admitting the conclusion." Then, after giving an example of the true syllogism, he says

there is this maxim resulting from it, "that whatever is predicated universally of any class of things, may be predicated in like manner of any thing comprehended in that class,"—the celebrated principle called the *dictum de omni et nullo* of Aristotle. After some observations on the substitution of letters and symbols for the terms of the syllogism, on apparent arguments, on the importance of finding a proper middle term, on generalization and abstraction, he winds up the analysis with the remark, "that it consists in referring the term we are speaking of to some class, *viz.* a middle term, which term again is referred to or excluded from (as the case may be) another class, *viz.* the term which we wish to affirm or deny of the subject of the conclusion."

With a very strong sense of the value of Dr. Whately's Elements,—of the correctness, and usefulness of the principles and views therein detailed,—it may be permitted me to observe, that even that work is, in some degree, deficient in the rigid

propriety of language which the subject demanded, and which might easily have been given to it. The analytical outline of logic can scarcely be regarded as a successful and complete elucidation of the science. Dr. Whately himself calls it an imperfect and irregular sketch.

For as the analytical outline, and the synthetical compendium, appear in juxtaposition, the reader naturally expects that they should answer exactly the one to the other, the analysis being the resolution of the whole into the parts, or, if the reader like it better, the tracing of given effects to the causes from which they spring;—the synthesis, — the enumeration of the several parts which combine to make the whole, or the advance from the cause to the varied effects or consequences. But this correspondence is by no means so clear as it might have been,—as it ought to be. For instance: having in the analysis stated that the operation of reasoning is in all the cases the same, (p. 25, fifth edition,) and that in every instance in which we reason a certain

process takes place in the mind, which is *one and the same* in all cases, Dr. Whately opens the compendium by saying, "There are *three operations* or states of the mind which are immediately concerned in argument." Again, after having in the analysis described the process in reasoning as the deduction of a conclusion from *two* other propositions, thence called premises, in the compendium he says, "Reasoning is the act of proceeding from *one* judgement to another, founded upon that *one*, or the result of it." These discrepancies may be more apparent than real; they may be of slight consequence: but the careful reader is, to a certain degree, distracted. And as the great object of the study of logic is to clear and to brace the mind,—as it is but the athletics and gymnastics of the reasoning faculties,—as clearness and strength are entirely dependant on perfect precision in the use of terms,—so the teacher of logic should avoid a verbal discrepancy as fatal to his science, as the man under training should avoid diluents and laxatives of every kind.

I am aware that some may think I have drawn too strictly the parallelism between the analytical and synthetical modes; but, after a careful perusal of Mr. Dugald Stewart's remarks upon the use of these terms in ancient and modern philosophy, showing that authority may be pleaded for using them in an exactly opposite and mutually convertible sense, and that he himself is at a loss to give them precise meaning,—sometimes confuting in the notes what he lays down in the text;—after reading also what Maclaurin says about these modes, in his account of Sir I. Newton's discoveries,—I cannot help considering them in a very simple and obvious light, as different or opposite modes of going over the same or a precisely similar path; according to the simile of Condillac, one being up and the other down the hill: only instead of saying, with Condillac, that as the two methods are contrary to one another, if the one be good the other must be bad, I rather say both may be good, according to the position and view which we assume, and the walk which, for the time, we

please to take. Dr. Whately, at any rate, cannot object to a rigid parallelism, since, in his Introduction, he uses the words as they are used in chemistry; and by synthesis he appears to understand the enumeration of elementary substances, thence proceeding onwards to simple combinations or more complex substances; by analysis only resolving these last, namely, the complex substances, step by step, into their simple elements. He should therefore obviously have begun his synthetical compendium with a clear and explicit enumeration of those very elements into which, in his analysis, he had resolved the science or art under review.

But the first and second sections of the third chapter and second book, on Arguments, contain a sufficient analysis of arguments, which are "*Reasonings expressed in words*;" and with such reasonings chiefly, if not entirely, is logic concerned. "An argument," says Dr. Whately, "is an expression in which, from something laid down and granted as true, (*i. e.* the premises,)

something else (*i. e.* the conclusion) beyond this must be admitted to be true as following necessarily from the other." Again, a syllogism,—(and let us remember that every just argument may be reduced into the form of a perfect or pure categorical syllogism, which is sufficiently obvious to the student of logic,)—"a syllogism is an argument so expressed that the conclusiveness of it is manifest from the mere force of the expression."

I conceive this to be a just and sufficient account of syllogistic reasoning, taken in connexion with what was before said respecting terms. By logic, the force of all reasoning, or the correctness of syllogism, is shown to depend entirely upon the degree of exactness and comprehensiveness of meaning in the terms employed, or, what is virtually the same thing, of distinctness in the things signified; and the chief purpose or use of the study is to call the attention of the mind to those forms of expression, and to fix those forms upon the memory, which are always essential to strict, legitimate, and convincing

inference. Now the value or correctness of a syllogism depends mainly upon one principal term, called the middle term.

"Every syllogism," says Dr. Whately, "has three, and only three, terms; *viz.* the middle term, and the two terms (or extremes, as they are commonly called,) of the conclusion or question. The middle term (called, by the older logicians, *argumentum,*) is that with which each of them is separately compared, in order to judge of their agreement or disagreement." Again he says, "Every argument consists of two parts: that which is proved, and that by means of which it is proved." And again, "The axioms or canons by which the validity of pure syllogisms is to be explained are these; *viz.* first, if two terms agree with one and the same third, they agree with each other; secondly, if one term agrees and another disagrees with one and the same third, these two disagree with each other."

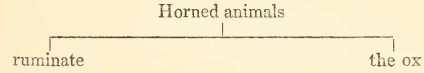
It is obvious that here we have but different views or statements of substantially the same thing, namely, the use of one

term, called a middle or third, as a means of comparison between two other terms; in other words, a reference of two terms to one and the same object of comparison, used as a measure or test of their agreement or disagreement,—in short, the celebrated *dictum* of Aristotle, that whatever may be predicated *universally* of a class of objects may be predicated of every individual comprehended in it; which is analogous to the axiom, or common notion of equality, that things which are equal to the same are equal to one another, or that the whole is made up of all the parts.

A syllogism, to make a homely simile, is a kind of two-pronged fork; the middle term is the handle which unites the prongs, and enables us to seize conveniently the object of our thought, and feed our mental appetite with food convenient for it.

E. g. All horned animals are ruminant; the ox is a horned animal; therefore the ox is ruminant. Supposing the habits of the ox, in particular, unknown or doubtful, but the circumstance of its possessing horns evident,

you infer that that may be predicated of the ox which you have previously predicated of the class to which it evidently belongs; and although the syllogism is evidently of little use in cases of experimental knowledge, where the validity both of the premises and the conclusion depends upon one and the same process of observation, yet is it evident that the first premise of the above example can only be true by involving the truth of the conclusion.



Whatever exhibits marks of design must have an intelligent creator; the universe exhibits marks of design; therefore the universe must have an intelligent creator.

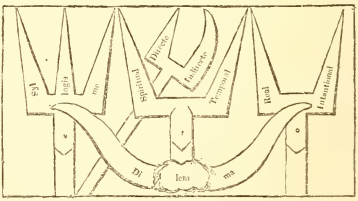
All reasoning is included in the term Logic; mathematics is reasoning upon figure and quantity; therefore mathematics is a branch of logic.

Logic

reasoning mathematics

The whole mystery of syllogism consists, therefore, simply in referring the two things whose relation you wish to ascertain, to some common class signified by what is called the middle term; or, regarding the conclusion as a truth you wish to demonstrate, the demonstration consists in referring it to some test, *i. e.* to some more general proposition, whose truth, whose use as a test, is previously admitted, agreed upon, or assumed.*

* My attention was arrested by the vignette attached to the original edition of Hobbes' Leviathan after writing the above, as exhibiting a similar idea of logic to the eye.



The connexion between logic and just classification

In examining the nature of syllogistic reasoning, it is now evident that you have,—

In the first place, something laid down, granted as true, or assumed as necessary, for the subsequent proceeding of the mind. It does not affect the truth of this position to determine whether you must always have two propositions, as in a regular syllogism, before a conclusion can be drawn; or whether the mind, having admitted one judgement or proposition, be led on from that one to another inevitably following from it. Suffice it, that in every argument you must begin with something laid down, granted, or assumed; in other words, you must have some datum or data, as points from which to start, or ground on which to rest.

Secondly. Having something laid down or granted, it is the characteristic of all cor-

proves that Hobbes is about right, though he meets with Mr. Stewart's particular reprobation, in saying that "when a man reasoneth, he does nothing else but conceive a sum total from addition of parcels, or conceive a remainder from subtraction of one sum from another," &c.

rect or logical reasoning, that the conclusion necessarily follows from the premises. It is involved in the premises; it is a consequence inevitably connected with them. An absolute necessity is the quality of all sound reasoning.

How then does logical or common reasoning differ from what we call mathematical or demonstrative reasoning? In both we have data from which to start, and conclusions inevitably resulting, involved in the meaning of the terms, *i. e.* in our conception of the things signified.

Will any man assert that logical reasoning is not demonstrative in the mathematical sense of the term? that is, does not convey a feeling of certainty to the mind in the justness and necessity of the conclusion. For my own part, I feel no difference in respect of absolute certainty. Having admitted that "whatever exhibits marks of design must have an intelligent author," and that "the universe exhibits marks of design," it appears to me as inevitable to admit that "therefore the universe must have an intelligent author," as to admit that the three angles of

a triangle are equal to two right angles, having admitted that the sum of the angles on the same side of a straight line, at the same point, is equal to two right angles. The conclusion is involved in the meaning of the terms: in the latter case it rests upon the notion of equality; in the former, the notion of design. It must be confessed, however, that the notion of design is not so immediately an *idea of sensation*, to use Locke's phraseology, as that of equality. It is an idea drawn rather from reflection, or what we are conscious of when the mind turns inward upon itself. In common language, two things are said to be equal when there are no sensible marks of difference between them. But an object of nature or of art will indicate design to an observer very much in proportion to the observer's power of appreciating the end aimed at, and the means employed; and surely it is impossible to separate the idea of design from the perception of *means* and *ends*.

It is not, then, in the circumstance of starting from certain data or given principles; it

is not in the necessity of the conclusion, as resulting inevitably from the data, that logical or common reasoning, and mathematical reasoning, differ from one another. We must seek for that difference elsewhere. When, therefore, the Edinburgh reviewer, after Kant, talks of necessary and contingent matter, as distinguishing two sorts of logic, one of which he equals to mathematical reasoning, the other to general reasoning, it is a talk without meaning, or at least without clear and sufficient meaning. For in all logic, as in all mathematics, the conclusion is equally necessary, equally contingent; equally necessary in the sense of inevitably following from, or being involved in, the data; equally contingent upon the comprehension or force of the terms; that is, the degree of clearness and exactness in the things signified.

But, again, if in all reasonings you start from some data, and in all reasonings you have necessary conclusions, where lies the difference between what we call mathematical and common reasoning? Obviously we must seek it in the nature or difference of

the data: and here, perhaps, we shall come to see in what sense it is true, if in any sense it be true, that in mathematics you have necessary, in logic contingent, matter. In mathematics we have, as I have shown, definitions, *i. e.* exact terms, significant of certain clear ideas of figure and quantity; and we are employed in tracing the relations of these ideas one with another. In all other reasoning we have also terms, and these terms are, or ought to be, signs of ideas. But while, in mathematical reasoning, we are concerned with ideas of figure and quantity, (to say nothing of forces and motion, with whatever else can be viewed and treated mathematically,) in logic we have every variety of term and of idea. There is no proposition of any kind, no number of words which can be put together so as to form a proposition, to whatever subject it may relate, which may not form part of a syllogism. It is owing to this comprehensiveness or vastness of logic, in its practical application, that its true nature is so little understood. In all reasoning, in all thought, as communicated from one mind to

another, we have, firstly, the terms; secondly, the things signified: but the sole question is, how far the terms are exact, and the things signified clear.

To put this in another light. If, as we have endeavoured to show, the essence of logic, of all arguments or syllogisms, consist in the reference of two terms to one and the same common term or object of comparison, used as a measure or test of agreement or disagreement; and if, as we have also shown, the essence of mathematical or geometrical reasoning consist in the perception of the agreement or disagreement of certain ideas and terms, by means of other intermediate ideas and terms, previously admitted as a test; it remains only to inquire what are the peculiar measures, or what is the excellence of the tests, in mathematical reasoning; for by these it obtains its character of force and clearness, that is, of demonstration.

Have you in mathematics better data, better measures, than in other subjects? The only answer is, that in mathematics you are conversant with ideas of figure and quantity,

and that you have certain well-defined terms always associated with the same simple ideas or impressions; the uniformity in the structure of the senses, and in the sensible impressions made upon them, laying the foundation for that peculiar constancy of language, and consequent clearness of reasoning, which we recognize in connexion with figures and numbers.

It follows, therefore, that it is not in any theories about generalization and abstraction, it is not in any difference between the higher faculties and the common faculties, that you are to seek for an explanation of the cogency of mathematical and the weakness of other reasoning. It is because your abstraction is so comparatively easy, and your generalization so complete; and because you have certain exact terms and symbols, used as known and admitted measures, or criteria of proof, about the application of which there neither is nor can be a possibility of doubt, that mathematical reasoning is so satisfactory.*

* Perhaps this comparative easiness of the abstraction and generalization in the mathematical sciences is the

If in that reasoning there were the slightest ambiguity in terms; if an angle were mistaken for a chord; if A B were mistaken for any other line than that intended in the geometrical proof; the reasoning would be vitiated, the demonstration lost, the link of concatenation broken; just as in arithmetic, if you mistook a 3 for a 5, your answer would be wrong; or, in more dignified phrase, your postulatum would exhibit an erratum. So, in logic, if the sense of your terms be changed in the premises and conclusion, the force of the syllogism is destroyed; the bolt of the stable is drawn, and the horse gone.

Think not there is no classification (and wherever classification is, there is of course

main thing intended by the reviewer when he speaks of the lower faculties only being employed and developed in the study; but the word "faculties" makes sad havoc in the writings of some philosophers, who would lead you to suppose man had as many distinct faculties as a centipede has legs. A faculty is simply a power; every action and every thought may be attributed to a distinct faculty for that action and that thought; and we may, if we please, talk of a faculty for sitting and a faculty for walking; but we gain confusion and lose distinctness by such phraseology.

abstraction and generalization,) in mathematics; for in the very first proposition you prove your triangle to be equilateral, when the sides are shown to belong to the class of lines which radiate from the centre to the circumference of the same circle, or of equal circles, of which class of lines you have premised equality as the characteristic; and he who should dispute this premise must look out for another proof of the first proposition than that with which his Euclid has furnished him.

It is to be observed particularly, that the definitions in geometry, and, I think, in all the branches of mathematics, are classifications or abstractions of a very simple kind: in geometry, of certain sensible impressions, derived partly from sight and partly from touch. Thus, a line is defined, "Length without breadth." It has been suggested that the words "without breadth" are unnecessary; but their use is perhaps expedient, in order to confine attention to that which is exclusively the object of the thought or reasoning. If any given line before us have

sensible breadth, that breadth forms no part of or consideration in the reasoning. Now more or less of what we call length is found in all visible and tangible objects. There is not an object which we see or handle that has not some outline, straight or curved. To this quality of objects, common to all, but peculiar to none, we give the name line or length. It is an instance of abstraction, simple and complete; and when we have lines of a certain kind, we give them a corresponding denomination, as straight, curved, waving. It is not however with lines, in this very general or abstract character, that geometrical reasoning has much to do; but it is concerned with lines in more precise and limited character; as they are related to each other in a certain fixed, perceptible, or conceivable manner: for instance, as perpendicular or inclined, as meeting in a point and forming angles, as parallel or equidistant, as forming squares, circles, ellipses, parabolas, or other curves, with fixed properties and relations. Hence figures are classed as triangles and parallelograms, as equilateral and equiangu-

lar, as pentagons, hexagons, polygons, and so on. And with regard to all these classes of figures, or definite arrangements of lines, the mathematical reasoning is strictly syllogistic; as in the fifth proposition of the first book, the proof of the equality of the angles at the base of an isosceles triangle turns upon bringing the angles in question within a certain class, *viz.* the class of angles subtended by equal bases, in triangles which have two sides of the one equal to two sides of the other, of which equality is demonstrated in the fourth proposition: and let us remember that every proposition in Euclid is demonstrated as true, not merely of the individual diagram before the student, but of its class, of which class the said diagram is, in respect of the reasoning, a perfect and sufficient example. Thus the angles of all triangles are equal to two right angles, whatever be the length of the sides, whether they be right-angled or obtuse; whether the lines be black or blue; whether it be the triangle on the paper, or a supposed triangle, formed by lines conceived to meet in the centres of the

earth, the sun, and the moon. *It is the simplicity and perfection of the classes;* the accuracy with which every term marks and defines the class; and the never-failing connexion between the terms and the sensible impressions, and the ease and certainty with which the sensible impressions lead to and support the mathematical conceptions and definitions; these things help to make, if they do not, as I conceive, themselves make, the proof so cogent and the assent so firm in geometry.

The dependence of the reasoning upon a clear apprehension of the definition, starting from it and adhering to it, becomes still more clear, if we look at the subject of proportion upon which Mr. Whewell has made some but not very distinct remarks. The fifth book of Euclid, which treats of proportion, Mr. De Morgan calls, in conjunction with Aristotle's logic, the most indisputable treatise that ever was written. On the other hand, Leslie, in the fourth Preliminary Dissertation to the Encyclopædia Britannica, says that it cannot possibly be

taught. The whole difficulty seems clearly to lie in the necessity of enlarging the mind's view of proportion, previously and strongly associated with numbers, *i. e.* with arithmetical proportion, to magnitudes whose relation to each other cannot be expressed in numbers; and this difficulty can only be overcome by the assiduous study of such magnitudes, and of a book, or books, in which such magnitudes and their relations are brought before the mind. The mind, by the study of the subject, grows to the apprehension of the definition, which is a general principle or view of a certain mutual relation of magnitudes, involving the truth of the propositions to which it is afterwards applied.

SECTION III.

HAVING now shown that the object of the science of logic is to call the attention to those forms of expression which are essential to valid arguments, in which the conclusion is necessarily involved in the premises, and the mind is led to perceive a connexion or relation which it did not before perceive between its ideas and terms,—that it resolves itself very much into just classification; having also shown that mathematical reasoning owes its clearness and cogency to the simplicity and clearness of its subject matter,—its abstractions and classifications, or relations of figures and quantities, being marked by defined terms, which are the media of mathematical proof,—we have to inquire how far other notions, besides those of figure and quantities, are susceptible of exact definition and exact language, and thereby of exact comparison one with another. This is the sum and substance of the question,—

the susceptibility of exact comparison between our notions of other subjects than figure and quantity; subjects less connected with sensible impressions, and in which our reasoning cannot be assisted or verified by an immediate appeal to the evidence of the senses.

I may be as certain, and doubtless I am, that there lived a celebrated orator named Cicero, at Rome, as that the angles at the base of an isosceles triangle are equal to one another; but it is evident that the ideas associated with the words Cicero, celebrity, oratory, Rome, are of a far more varied and complex character than the ideas associated with the terms of the above or any mathematical proposition; and of a hundred persons who will equally readily assent to the historical or moral proposition, the ideas associated with its terms will differ by a thousand modifications and varieties. Here, then, lies the difficulty. Forms and magnitudes visible to the eye, and weighed by the hand, can be compared, and their exact difference can be estimated and described. But who shall compare and

estimate the exact difference between two tastes and two sounds? and how indefinite must those terms remain which are not only associated with a number of such sensible impressions incapable of comparison, but with trains of sensation and emotion, the traces of which pass away with the moment in which they have birth, and which are, perhaps, wholly similar in no two existing minds.

It is often said, by way of distinguishing mathematical science from all other kinds of reasoning, that the mathematics are human contrivances for attaining human ends; and that in the reasoning we are only evolving the nature and properties of these contrivances. Thus in numbers, we have fractions and decimals; in algebra, equations and roots; so in the affairs of life we have invented common measures with which to compare magnitudes and quantities: thus we have the foot, the yard, the pound weight, the ton, the pint, the bushel; by means of which quantities, distances, and magnitudes, are relatively ascertained and settled. So we have

thermometers, barometers, and chronometers, for measuring atmospheric weight, heat, and time. These are some of the received measures or tests for ascertaining and determining the degrees of difference by which one quantity varies from another, of whatsoever can be measured or tested.

Let us, however, remember that all language is a human contrivance for expressing human thoughts in all their wide relations and variety; and although the language which we daily use may have, and most general terms have, very different ideas associated therewith in different minds, yet *reasoners* have a certain power over its use. We can examine, control, and the exact reasoner always seeks to control by examination and reflection, the associated ideas. It should seem, indeed, that where we have no common measures or exact standards of comparison, we cannot come to any exact conclusions. And this must be the case in all questions of degree, in things that cannot be reduced to measure; where we must be content to use our comparatives and superlatives

indefinitely, and satisfy ourselves with the old maxim, 'De gustibus non disputandum.' *But it is in our power to approach to exactness, (in mathematical phrase, as nearly as we please,) by using well-defined terms associated with uniform sensible impressions*, and by distinguishing things that differ in *kind*, if not in degree. We may scrupulously avoid unnecessary changes of terms, when the subject matter of our reasoning is the same; leave nothing to be understood which it is possible to express; and beware of using relatives and pronouns, to which there is no clear or certain antecedent.

It is also often said, by way of further distinguishing the metaphysical sciences from the mathematical, that the latter turns upon human abstractions or hypotheses, and you begin with definitions; whilst the former turns upon facts, and you end with definitions. But is this an adequate distinction? Every one, doubtless, must be aware of a certain difference between what are called inquiries into facts, and the pursuit of a train of mathematical reasoning. But what is that

difference? What is the process of mind in the two cases? In mathematical propositions you have all the facts before you, and are deducing logically the consequences which flow from the acknowledged data. In what are called inquiries into fact, you are generally testing the correctness of some verbal statement, which indicates the existence of some but perhaps by no means clear impressions upon the human mind; or you are seeking to supply the want of impressions on your own senses, by considering the nature and evidence of the impressions on the senses of others. These impressions you gather from what is called testimony; and testimony introduces all the ambiguities and difficulties of language. Here you must have various methods of examination or tests of truth drawn from experience and suited to the particular case under examination, into which it is unnecessary to enter at length.

It is often difficult, we say, to ascertain what is the fact. In general all the difficulty arises from, or is increased by, the indefinite-

ness of the terms in which the statement of fact is enwrapped by the witnesses, and the want of care, caution, and skill in observing and registering observations. The extreme rapidity with which the human mind mingles its own inferences with its observations or sensible impressions; the confidence with which it attaches imaginary or experienced causes to the perceived effects, and the difficulty of dissociating the one from the other, give rise to various embarrassments not merely with regard to reliance upon the statements, but in ascertaining even the meaning of others,—embarrassments which most writers of reputation in the abstruse subjects of morals and religion rather increase than help to overcome. However, "it is possible," says the sweet-minded Hartley, "for two persons of intelligent and candid minds to understand one other."

Then, as to ending with definitions, the metaphysician who talks of that as his end, while it is the mathematician's beginning, should remember that this is only another

mode of stating that his metaphysical discussions are disputes about words. As such no doubt they have their importance, the question being, What are the ideas associated with, or the mental phenomena to be classed under, certain terms which exercise extensive sway over the thoughts, feelings, and conduct of men? Let the metaphysician, then, take care of the road by which he proposes to pursue this end. Let him take heed whilst he is in pursuit of a proper definition of a certain term, of such a definition as will be acceded to by the student, that he does not embarrass himself and his reader by a multitude of other still less defined or definable expressions, which thicken the darkness and the difficulty, and conduct to dismay and to despair.

These simple observations will, I conceive, go some way towards illustrating the question, in what manner, if in any manner, demonstrative reasoning may be connected with and obtained in metaphysical and moral science. Logic calls our attention to the nature and force of terms; to those

forms of expression which are essentially connected with valid reasonings — reasonings to which the mind assents through the very nature of the terms. Mathematical science exhibits the perfection of such reasoning and such terms. It works with symbols significant of certain clear and uniform notions of figures and magnitudes. In its higher branches it is indeed the very science of symbols; its results arising out of the conceived and admitted nature of the symbols themselves, and their uniform relations one with another.

Before saying a few words on the application of demonstrative reasoning to physical and metaphysical science, I cannot forbear remarking how groundless is the common notion that demonstrative reasoning is stronger than all other kinds of evidence,—than the evidence of the senses or of testimony;* a

* There is some logical impropriety in connecting the words demonstration and evidence, and in talking of demonstrative evidence. In demonstrative reasoning we trace the harmony subsisting among our agreed principles or admitted notions and conceptions, and the consequences or connected notions. The senses (and testi-

notion which makes some men call for demonstration where it is evidently absurd to make that call, unless it be remembered that the nature of every proposition must determine the nature of the demonstration, or the kind and criteria of proof. But who shall undertake to demonstrate by any admitted criteria of proof that milk is white and the sky sometimes appears blue? These are simple instances of names attaching to the objects of sense, and all communion of mind depends on agreeing to give similar names to similar impressions. And what folly would it be when my servant testifies that a crowd is in the streets, or Mr. Smith is in the study, to ask him to demonstrate these affirmations and propositions. The conduct of life depends upon notions and habitual statements which require no demonstration, no laboured process of inquiry and proof; and with the general language of human intercourse, the associated ideas are

mony, which must be ultimately referred to impressions on the senses) *evidence* the existence of certain causes of sensation external to ourselves, independent of our own minds, and supply the fundamental conceptions from which we reason.

sufficiently clear and uniform for the harmony and happiness and the ordinary wants of society. But when men aspire to what is called science, then must they take heed of language, as the ladder on whose rails the foot must rest in every step of the ascent.

Now with regard to physical science, it is obvious man is simply the observer and registrar of external nature. "Homo est naturæ minister," is the short and obvious maxim of the natural philosopher, and it is evident that the sciences which we call inductive merely give us the combined results of human observation in various specified departments, arranged, classified, and marked, and mathematical science is essential to perfect our observations.

As we range the walks of time and space, every new object or relation which presents itself to attention, a new term, significant of that object or relation, must be given. That same term will suffice for the same object or its counterpart, when the mind meets with it again. And the uniformity and variety in nature gives rise to the uniformity and variety in the structure and use

of human language, which, as a whole, may be considered a mystical radiation from nature, imprinting its pictures upon the subtle ground-work of the mind. A term becomes general by being applied to many similar objects or impressions. The uniformity in the objects and processes leads us analogically to apply the term laws to the phenomena and processes of nature, as science in its various departments reveals and registers the order, succession, and character of these phenomena. Each science having its own peculiar phenomena to register, requires its own peculiar terms; nor can we, where new objects of perception, new relations of thought, are to be expressed and held forth to the contemplation of the mind, object to many new and therefore hard terms. But in these days the student of nature must be warned against supposing that a mere knowledge of terms is scientific knowledge, although it must be admitted as an indispensable and an unavoidable part thereof. The learner of a new science cannot but be as the child, whose

understanding grows to the meaning and right use of the language of the world around him, nor must he complain of difficulties and impediments. Happy the learner, however, who is in the hands of judicious guides, who consider terms subsidiary to instruction in things, with whom books are, as they assuredly ought to be in physical science, not the substitutes for the companionship of nature, but the aids to interpret her lessons, and to observe and to arrange her instructions. The botanist must keep in the fields and the garden; the chemist in the laboratory; the geologist in the quarry, by the hill side or under the cliff; the astronomer must sweep the heavens with his glass, and report to others

> "Of fields of radiance, whose unfading light
> Has travell'd the profound six thousand years,
> Nor yet arrived in sight of mortal things."

And it is the beauty of physical science, when legitimately and lovingly pursued, that it calls us into communion with the Creator as he reveals himself in his works, and away from the perverse disputes and vain jang-

lings of men. In physical science there is comparatively little of tiresome useless argumentation; the facts on which the classifications and conclusions rest are evident to those senses of whose use not even the Fall has deprived the children of Adam. And though the senses may sometimes deceive us from a kind of natural difference in keenness or constitutional imperfection, or through hasty inferences and casual associations, yet the philosopher who builds his system of science, physical or metaphysical, upon other ground, who thinks the root of the tree of life and knowledge is not in that plain but all-supporting soil, had better return at once to the speculations of the schoolmen, and puzzle himself with inquiries into the necessary attributes of spirits that have never inhabited a body, or dilate upon that pleasant and edifying subject, "chimæra bombinans in vacuo."

Vanity, partiality for their own habits of study, and their own modes of classification, may even in physical science lead men to differ widely and warmly; but the differ-

ence will hardly be very sore, if unconnected with any worldly interests. Questions of fact, and questions of the meaning of terms in most matters of pure science, would be settled without length of debate, if men did not wish to appear wiser than they are; if they were content patiently to learn and mildly to instruct; if they sought the knowledge and love of nature rather than the estimation of men; if they looked upon themselves as mutual interpreters, and mutual servants of the will of *Him*, who has made of one blood all nations of the earth to dwell together upon its face, and bound the vast family of man together by the strongest ties of mutual interest, making it their chief and noblest happiness to benefit and assist each other.

In the pursuit of physical science it would be very easy to repeat some of the principal rules to be observed; but they are already in various forms and in abundance before the studious and inquiring world. To observe and register carefully, not to generalize too fast, to take special care of your premises before you arrive at your conclusions, to build the super-

structure of system upon the solid groundwork of clear and well-ascertained facts, and with the strong masonry of well-defined and chastised language—the cement of human knowledge—these are the short and simple, but universal rules for all philosophy. The Novum Organum of Bacon; the first of the preliminary dissertations prefixed to Buffon's Théorie du Monde; the admirable Reflexions sur la Geométrie, in the Pensées de Pascal; Hartley's invaluable Observations on Propositions and the Nature of Assent; the last book of Locke's Essay on the Human Understanding; Herschel's Treatise on the Study of Natural Philosophy; Dr. Whately's Logic; some of the remarks of Laplace, in his Essay on Probabilities, to which I shall hereafter advert, and of Cuvier in the Preface and Introduction to his Animal Kingdom; and Mr. De Morgan's treatises published by the Society for the Diffusion of Useful Knowledge;—are among the works of most importance in connexion with the subject of this Essay. The following passage from the preface to Cuvier's first edition of his Règne

Animal will be acceptable to the reader who is not previously acquainted with it, and bears closely upon our present course of thought. "The habit necessarily acquired in the study of natural history, of the mental classification of a great number of ideas, is one of the advantages of that science which is seldom observed, and which, when it shall have been generally introduced into the system of common education, will become perhaps the principal one. By it the student is exercised in that part of logic which is termed method, just as he is by geometry in that of syllogism; because natural history is the science which requires the most precise methods, as geometry is that which demands the most vigorous reasoning. Now this art of method, once well acquired, may be applied with infinite advantage to studies the most foreign to natural history. Every discussion implying a classification of facts, every inquiry which demands a distribution of materials, is performed according to the same laws; and the young man who had cultivated this science merely for amusement, is surprised,

when he makes the experiment, at the facilities it affords him in disentangling all kinds of affairs. It is not less useful in solitude. Sufficiently comprehensive to satisfy the most powerful mind, sufficiently various and interesting to calm the most agitated soul, it consoles the unhappy, it soothes animosities. Once elevated to the contemplation of that harmony of nature irresistibly regulated by Providence, how weak and insignificant appear those causes which it has been pleased to leave dependant on the arbitrary will of man. How astonishing to behold so many examples of fine genius consuming themselves so vainly for their own happiness, or that of others, in the pursuit of empty speculations, whose very traces a few years suffice to sweep away."

But is the motto "homo est naturæ minister" applicable solely to physical and not to metaphysical, moral, and political science? Can we study the human mind and human interests without a careful observation of the phenomena or facts? Is man to interpret his mental and moral constitution

by means of some *à priori* principles, which are to be received, without question, upon the ipse dixit of this or that philosopher? There is indeed a growing perception of the importance of the Baconian method, in moral and political science as well as in other departments of human inquiry, notwithstanding the tendency of many productions of eminence to draw away men's attention from the palpable instruction of nature to the cloudy, obscure, inconsistent, and unmeaning language of the schools. But a slight acquaintance with the most popular works on ethical and metaphysical philosophy, will lead to the conclusion that they are sadly deficient in that exact language which is the essential characteristic of all true science, and must be its foundation.

"A plain and unadorned style," says Lord Bacon, "is the proper style for philosophy." Yet how far from being plain,—how studied in variety of phrase,—how redundant in poetic and metaphoric ornament, are the pages of a Stewart and Mackintosh! Their pages seem to have been written under

the impression that no effort of art should be spared to beguile the reader in the study of matters so dry and so laborious as ethics and metaphysics. But the result of this is to give the real student, who is carefully looking out for facts and principles, more trouble than can be estimated in finding the object of his search, — in separating the important from the trivial,—in testing, by the application of precise logic, the presence and quantity of meaning in a cloudy solution of verbiage. Their dissertations leave hardly any definite impression on the mind. Nor is this owing to any peculiar difficulty in coming at the facts which lay the foundation for principles, so much as to an erroneous view of the true method, or from a bad habit of philosophising, or perhaps to an indisposition for that patient and cautious examination of details, with that simplicity and exactness of phrase, which have few charms for any but the obscure devotees of truth.

Metaphysical discussions are pre-eminently discussions about words. Ideas, feelings,

principles, faculties, powers, affections, materialism, spiritualism, necessity, free-will, *cum multis aliis*, occur so constantly in these discussions, with so little exactness of meaning, and such variety of indistinct associated notions, that the student is still, like Milton's angels, "in wandering mazes lost."

The dissertations of Stewart and Mackintosh, prefixed to the Encyclopædia Britannica, enjoy a high degree of reputation. They are spoken of by critics of eminence as inimitable and invaluable. Reviews of great authority delight in recording the highest estimation of their merit. They are unquestionably useful and pleasing; but of what are they the history? If we compare them with the accompanying dissertation by Playfair, on the History of Natural Philosophy, they must sink in the comparison, inasmuch as they fail to give that systematic view of real though gradual additions to human knowledge,—of discoveries, with the authors, and times of discovery, which can alone properly constitute a history of any science or philosophy. They do not fasten upon im-

portant epochs of improvement in the mode of studying and treating the philosophy of mind. They do not give the details of writers and their systems, in the clear and exact manner most essential to the reader, nor sum up their respective merits and defects by the application of any well-stated, established, and acknowledged principles or criteria of excellence.

Mr. Stewart makes the chief merit of Locke to consist in the inquiring spirit with which he imbues his reader. Speaking of the general effect of Locke's discussions, in preparing the thinking part of his readers to a degree till then unknown, for the unshackled use of their own reason, he says, "This has always appeared to me the most characteristical feature of Locke's Essay; and *that* to which it is chiefly indebted for its immense influence on the philosophy of the eighteenth century." The Essay of Locke is certainly the production of a profoundly thoughtful mind; and the diligent reader, called into sympathy with such a mind, must partake, and imbibe in some degree, the spirit

of the great and good philosopher. But surely the merits of Locke, as the founder of a school of metaphysicians, are of a much higher order than such an estimate of his chief characteristics implies. Locke not only prepares the reader to reason, but prepares him to reason well; to know himself, and man, and nature, better. It is difficult for us in these times to conceive the clouds and mists which the Essay on the Human Understanding dissipated, and how it illuminated the whole horizon of human thought. Cudworth's Intellectual System is a book which sets the reader upon using his own reason; and it requires a vastly superior stock of erudition to weigh and consider its contents. But Cudworth's learning oppresses the mind with its weight, and lies upon the intellect like a London fog upon the chest; whereas Locke clears and braces the understanding. He has laid the broad and deep basis for a true intellectual system in his grand division of Sensation and Reflection; the world without us, and the world within. His second chapter, on the Origin of our Ideas, and his last

chapter, on Knowledge, Reason, Faith, and Judgement, are full of rich, just, and invaluable matter. In the 28th chapter of the 2nd book he has given, incidentally, an analysis of moral laws, which must be in part followed by every other inquirer into ethics and jurisprudence; and the few words of Mr. Austin (in a note, p. 174 of his work on Jurisprudence) are far more worthy of the subject than all Mr. Stewart's elaborate though elegant phraseology. "Allowing for defects, which were nearly inevitable, his analysis is strikingly accurate. It evinces that matchless power of precise and just thinking, with that religious regard for general utility and truth, which marked the incomparable man who emancipated human reason from the yoke of mystery and jargon. And from this, his incidental excursions into the field of law and morality, and from other passages of his essay wherein he touches upon them, we may infer the important services which he would have rendered to the science of ethics if, complying with the instances of Molyneux, he had examined the subject exactly."

Mr. Stewart's estimate of the defects

and excellencies of Locke can never satisfy the deep and cautious investigator of the constitution and powers of the human understanding; and his everlasting, untiring intimations that the principles of Locke's Essay, if carried out, or not guarded by Locke's admissions, lead to the scepticism of Hume, are beyond measure unsatisfactory and wearisome to those who neither see nor understand the consequence, and who find it difficult to make out, at least from Mr. Stewart's statements, in what the scepticism of Hume precisely consisted, or how far it extended, save in the celebrated Essay on Miracles, where the uniformity of nature, or our own experience of it, is contrasted, in a loose manner, with the variableness of human testimony, and perhaps in a general want of principles and convictions in all the objects of metaphysical inquiry. There are two very different states of mind often confounded under the term scepticism; one is that of an inquirer, who is desirous of placing his own meaning, and the meaning of others, before him in the clearest and most satisfactory light, and pro-

portioning the degree of his assent to the strength of the evidence; the other, that of a man who concludes positively against certain opinions and propositions, as having no foundation in truth, merely because he cannot keep steadily before him the chain of arguments or evidence by which they are supported. The former is allied to what Hartley calls a "religious scepticism," which receives nothing rashly, and is always prepared and seeking for evidence, with a view of concluding wisely and rationally. The latter is less allied to scepticism than to dogmatism, and becomes dogmatism when a man, losing sight of the degrees of evidence by which the opinions he controverts are supported, thinks to bear them down by the strength, rather than the justness, of his assertions. It is that of a man who is not seeking for truth, but doubts its existence; who is in the habit of questioning every thing, and believing nothing.

But if Mr. Stewart's work be open to criticism, what shall we say of Sir James Mackintosh, whose Dissertation excites the prin-

cipal reviews to rapture, and which Mr. Whewell undertakes to preface and to edit? That it is well—very well—worth perusal, may be asserted, partly because so little is written and so little is read with becoming care in such departments of human inquiry. But that the more it is read and examined, the less satisfactory it will appear, may also be asserted; and he who does not choose to rest his justification of this assertion upon any elaborate showing of his own, may shelter himself behind Mill's Fragment on Mackintosh, and ask, Where is the answer to that book? He may ask why the laudatory reviewers never allude to its existence, and why, since it contains so vigorous, hearty, and relentless an attack upon the merits of Sir James, there is no attempt to protect his fame, to meet and rebut its arguments. Surely Mr. Mill was no puny adversary—no mere fly, that stings but impedes not the noble racer in his dazzling course. The peaceful student of nature and of truth may not admire the somewhat bearish style in which Mr. Mill shakes and tears his prey; and with his Fragment strews the

ground with fragments of Sir James's mangled Dissertation. He may feel that the arrow from Mr. Mill's quiver, when he touches upon Sir James's "dandy philosophy" and lack-a-daisical style, has been dipped too cruelly in poison, tinged too darkly with the "gall of bitterness." Yet while his feelings of pity and tenderness are roused, and he cries out for mercy! mercy! on Sir James, it is a cry for mercy rather than for justice. The soft heart wishes to spare, and not to trample on the fallen. But Mr. Mill, roused by the apparent, though as he calls it affected candour, with real injustice to Mr. Bentham, descends like a second Achilles to avenge his friend, and drags his victim thrice round the field of his defeat.

Seriously, he convicts Sir James over and over again of those superficial statements, of that vagueness and confusedness of language, which have hitherto been, and which continue to be, the great impediments to sound progress in the philosophy of mind. Yet, to do Sir James some justice, he is free from that aversion to acknowledge the full

extent and importance of the "law of association," which is characteristic of many, I believe most, of the Scotch metaphysical writers. He shares not in Dugald Stewart's antipathy to Hartley. He is the first metaphysical writer of considerable repute of late years, as far as I know, who seems to be somewhat duly aware of the merit and importance of the Hartleian theory and principles. He is wrong when he affects to see deeper into the moral constitution of man than others who have gone before him. He is convicted of error and absurdity in supposing that those of whom he was writing had overlooked a distinction which he had sagacity to discover between thoughts and feelings, perceptions and emotions. And even while he admits that Hartley was a great philosopher and a good man, there is hardly a sentence upon the nature of Hartley's system which conveys to the reader a distinct idea of it,—of its apprehended merits or defects. "Nothing," says Sir James, "more evidently points out the distinction of the Hartleian system from all systems called self-

ish, not to say its superiority in respect to disinterestedness over all moral systems before Butler and Hutcheson, than that excellent part of it which relates to the Rule of Life." Now the Rule of Life contains the whole moral system of Hartley, and is a deduction from the inquiry which Hartley instituted into the frame and constitution of man. And in the whole compass of human philosophy does any thing surpass that rule of life? But for Butler and Hutcheson, it is evident that Hartley would have been the favourite philosopher of Mackintosh. Now since, according to his own showing, Butler and Hutcheson have really no system, but merely assert that human nature has certain characteristics, which no one ever denied, and which the philosophers of the Hartleian school have resolved, or tried to resolve, into the true elementary principles, we may perhaps, notwithstanding Mr. Mill's caustic animadversions, admit that the Dissertation of Sir James Mackintosh will, after all, help the student of mental philosophy to the quarter where light is to be found. But he

must continue to be a student, and verify statements and weigh opinions for himself.

It may appear unwarrantable to some readers that I should have lapsed from an inquiry into the nature of mathematical reasoning and its application to science, suddenly to a criticism upon the Dissertations of Stewart and Mackintosh. But the reason is obvious: these Dissertations appear to be the most popular, and so far the most important, treatises published of late upon the mental and moral constitution of man, which is doubtless the noblest of all human objects of inquiry and contemplation. I see or fancy that these Dissertations, both admitting and lamenting the ambiguity of language as the greatest obstacle in the way of the moral philosopher, are utterly deficient in earnest and successful efforts to grapple with the difficulty and overcome it; nay, increase rather than diminish the evil. True it is, that while we confine ourselves to that which we can rigidly and logically prove, in the present state of ethical science, we shall go but a little way, and we shall

soon come to all the conclusions which can be safely concluded, whether gathered from our most comprehensive inductions or flowing from the nature of our conceptions and terms.

> ——————" How little can be known!
> This is the wise man's sigh: how far we err!
> This is the good man's not unfrequent pang."
> *The Excursion.*

But it is better to make sure of a little than to darken counsel by words without knowledge; it is better to build a small house fit and furnished to dwell in, than ruin our fortunes by undertaking a palace which we can neither finish nor inhabit; and let us remember that philosophy is, after all, only strong sense, or the combined result of larger observation and deeper reflection in a little better dress than that of ordinary life. When a writer or reader sits down to philosophize, or to logical composition, he should consider first what it is he proposes to prove; and secondly, by what means he proposes to prove it; and he will soon find that he must make the nature and use

of language his principal study, as it is the main instrument of thought and communication; and in the proper use of it, consists the chief difference between the strong and the weak mind.

"Scribendi recte sapere est et principium et fons."

In considering the application of mathematical reasoning to moral and metaphysical subjects, it would be unpardonable to overlook the Philosophical Essay of Laplace, on Probabilities, reviewed by Playfair in the twenty-third volume of the Edinburgh Review. Laplace has touched upon the applicability of the calculus to moral sciences in the following order of topics:—First, the probabilities of testimony; second, the votes and decisions of assemblies; third, the judgements of legal tribunals; fourth, tables of mortality, the mean duration of life, of marriages, and any social relations; fifth, illusions in the estimate of probabilities.

Although many just and some curious and novel observations are made by Laplace under these heads, yet their general effect is

to show the wide difference which exists between those events which, by a strict and clear resemblance, can be classed together and subjected to numerical calculation, and those which are of more complicated character, have a less perfect analogy, differ from each other by many circumstances peculiar to each case,—diversities growing out of the general mental, and moral constitution of man, and which cannot, therefore, be so legitimately classed, nor so successfully brought within the category of number. It is indeed somewhat singular, that Laplace should have placed tables of mortality and the duration of life, and the relative proportion of births, marriages, and deaths, to population,—events of such uniformity of character, and so easily and naturally classed,—in connexion with subjects so wide and general as the nature of testimony, the decisions of popular assemblies and legal tribunals, and the illusions of imagination,—subjects which he has touched but slightly. Statistical calculations ought not indeed to be overlooked by the moralist,

since they furnish that extended view of the phenomena of life and of the moral world, which is essential to the deduction of sound moral rules and principles, and affords great help to the interpretation of the laws and ends of Divine Providence in the administration of the world. Any collection of analogous phenomena in the physical world is of importance in the moral, since such collections lay the foundation of general rules, and supply the exact knowledge which is of the greatest use in the conduct of life. But in collecting analogies we must not overlook differences, and be duly cautious in the application of general maxims, which are apt to mislead as well as to instruct. Playfair has made some just remarks on the vagueness of Laplace's language and views on the subject of testimony; and it is easy to conceive that Laplace's devotedness to the exact sciences may have disposed him to an unreasonable mistrust and disregard of all conclusions of the pure intellect, drawn from the phenomena of history and the moral world. But when Playfair intimates that no

conclusion founded on the application of the calculus to moral probabilities should be allowed to interfere *with the truths of religion*, he seems to imply that such an interference may be anticipated, and rationally feared, and to forget that, to the philosophic mind, truth in all departments must be ever consistent with itself; that though the atoms of the universe may be weighed and measured, and every seeming accident shall be reduced to order and to rule, the reasons for adoring the Creator, and trusting in him and obeying him, and for loving our neighbour as ourselves, will only be proportionably multiplied.

It is to me truly interesting to observe Laplace perceiving and recording the importance of that principle of association into which Hartley, followed by Mill, resolves all reasoning. "The most fertile of all the principles of psychology," (Laplace, p. 224, of the 8vo. edition of his Essay,) "is that of the association of all things which have had in the sensorium a simultaneous or regularly successive existence; an association whereby

the return of one calls up all the others connected with it; the objects which we have formerly seen awaken the traces of things which in the first view were associated with them. These traces call up in the same manner those of other objects, and so on in succession, so that by means of one thing presented to the mind, we can recal an infinity of others, and rest our attention upon whatever we wish to consider. To this principle the employment of signs and language for recalling sensations and ideas belongs; it accounts for the formation and analysis of complex, abstract, and general ideas, and for reasoning. Many philosophers have well developed this principle, which up to the present time constitutes the real part of metaphysics." "I have endeavoured to show in these papers," says Hartley, at the conclusion of his 99th proposition, "that all reasoning, as well as affection, is the mere result of association."

Some further extracts from Laplace might perhaps be given with interest to the reader, but it does not appear to me that Laplace

adds any thing to the ingenious observations and speculations of Hartley, in his chapter on the Deduction of Rules for the ascertainment of Truth, and advancement of Knowledge, from the mathematical Methods of considering Quantity. Mr. Dugald Stewart, in the Preliminary Dissertation to his Philosophical Essays, was pleased to sneer at some of the most refined and natural conclusions of that great and good philosopher, Hartley, in the chapter referred to, and to consider them as evidence of the unsoundness of his understanding. But Mr. Stewart is very often unfortunate in his selection of the sentiments of the greatest English metaphysicians, which he marks for peculiar reprobation and contempt.

I have now brought together, and laid before the reader, some thoughts which have occurred to me when reading the more popular scientific publications of the day. I am not without hope of awakening the attention of a few, into whose hands these pages may fall, to the contents and merits of authors whose works they may not already suffi-

ciently appreciate, but which it is good "to feed upon, as insects on a leaf, till the whole heart be coloured with the fibre." The reader who wishes to see one of the best examples which our times have afforded of strictly logical or demonstrative reasoning applied to moral and metaphysical subjects, will do well to study the work of Mr. John Austin, if he is not already acquainted with it, entitled, "The Province of Jurisprudence Determined," being the substance of lectures delivered at the University of London as Professor of Jurisprudence. Contrary to the practice of those numerous writers who confound that of which they are treating with every thing else with which it can, by possibility, be confounded, Mr. Austin carefully distinguishes his subject from every thing with which it is, howsoever remotely, related. He is one of the few writers who does not fear to repeat himself, so much as to be misunderstood by his reader. He would rather give you a whole sentence two or three times over, than introduce a pronoun whose antecedent might be doubtful or nonexistent. He seeks not, by

showy phrase, to impose on the understanding of his reader, or to smooth over with received verbiage the difficulties of his subject and the absence of thought; and if he sometimes wearies with the repetition of a lengthened phrase, for which a convenient abbreviation might have been advantageously adopted, we could forgive much more than this to a writer who feels so acutely the audacity of the paradox, "that men really should think distinctly and speak with a meaning." As it bears very closely, indeed, upon the topic discussed in these pages, and as the matter is itself of the highest importance in connexion with ethical inquiries, I shall not scruple to close this essay with the following extract from Mr. Austin's work, feeling, as I do, its fullness of truth :—

"If there were a reading public numerous, discerning, and *impartial*, the science of ethics, and all the various sciences which are nearly related to ethics, would advance with unexampled rapidity.

"By the hope of obtaining the approbation which it would bestow upon genuine

merit, writers would be incited to the patient research and reflection which are not less requisite to the improvement of ethical than to the advancement of mathematical science.

"Slight and incoherent thinking would be received with general contempt, though it were cased in polished periods, studded with brilliant metaphors. Ethics would be considered by readers, and therefore treated by writers, as the matter or subject of a science; as a subject for persevering and accurate investigation, and not as a theme for childish and babbling rhetoric.

"This general demand for truth, (though it were clothed in homely guise,) and this general contempt of falsehood and nonsense, (though they were decked with rhetorical graces,) would improve the method and the style of inquiries into ethics, and into the various sciences which are nearly related to ethics. The writers would attend to the suggestions of Hobbes and of Locke, and would imitate the method so successfully pursued by geometers; though such is the variety of the premises which some of their

inquiries involve, and such are the complexity and ambiguity of some of the terms, that they would often fall short of the perfect exactness and coherency which the fewness of his premises, and the simplicity and definiteness of his expressions, enable the geometer to reach. But though they would often fall short of geometrical exactness and coherency, they might always approach, and would often attain, to them: they would acquire the art and the habit of defining their leading terms; of steadily adhering to the meanings announced by the definitions; of carefully examining, and distinctly stating, their premises; and of deducing the consequences of their premises with logical rigour. Without rejecting embellishments which might happen to fall in their way, the only excellencies of style for which they would seek are precision, clearness, and conciseness; the first being absolutely requisite to the successful prosecution of inquiry, whilst the others enable the reader to seize the meaning with certainty, and spare him unnecessary fatigue.

"And what is equally important, the protection afforded by this public to diligent and honest writers, would inspire into writers upon ethics, and upon the nearly-related sciences, the spirit of dispassionate inquiry, the indifferency or impartiality in the pursuit of truth, which is just as requisite to the detection of truth as continued and close attention, or sincerity and simplicity of purpose. Relying on the discernment and the justice of a numerous and powerful public, shielded by its countenance from the shafts of the hypocrite and the bigot,—indifferent to the idle whistling of that harmless storm, they would scrutinize established institutions and current or received opinions fearlessly, but coolly, with the freedom which is demanded by general utility, but without the antipathy which is begotten by the dread of persecution, and which is scarcely less adverse than 'the love of things ancient' to the rapid advancement of science.

"This patience in investigation, this distinctness and accuracy of method, this freedom and 'indifferency' in the pursuit of the

useful and the true, would thoroughly dispel the obscurity by which the science is clouded, and would clear it from most of its uncertainties. The wish, the hope, the prediction of Mr. Locke, would, in time, be accomplished, and 'ethics would rank with the sciences which are capable of demonstration.' The adepts in ethical as well as in mathematical science would commonly agree in their results; and as the jar of their conclusions gradually subsided, a body of doctrine and authority to which the multitude might trust would emerge from the existing chaos."

FINIS.

R. KINDER, Printer, 21, Great New Street.